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NMHDECAY: A fortran code for the Higgs masses, couplings and decay widths in the NMSSM

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Abstract: The Fortran code NMHDECAY computes the masses, couplings and decay widths of all Higgs bosons of the NMSSM in terms of its parameters at the electroweak (or Susy breaking) scale: the Yukawa couplings λ and κ, the soft trilinear terms A_λ and A_κ, and $\tan\beta$ and $\mu_{\rm eff} = \lambda\langle S\rangle$. The computation of the spectrum includes leading two loop terms, electroweak corrections and propagator corrections. The computation of the decay widths is carried out as in HDECAY, but (for the moment) without three body decays. Each point in parameter space is checked against negative Higgs bosons searches at LEP, including unconventional channels relevant for the NMSSM. One version of the program uses generalized SLHA conventions for input and output.

Keywords: Supersymmetry Phenomenology, Supersymmetric Standard Model.

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1. Introduction

The Next to Minimal Supersymmetric Standard Model (NMSSM [1]–[11]) provides a very elegant solution to the μ problem of the MSSM via the introduction of a singlet superfield $\widehat{S}$. For the simplest possible scale invariant form of the superpotential, the scalar component of $\widehat{S}$ acquires naturally a vacuum expectation value of the order of the SUSY breaking scale, giving rise to a value of μ of order the electroweak scale. The NMSSM is actually the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY breaking scale only.

In addition, the NMSSM renders the "little fine tuning problem" of the MSSM, originating from the non-observation of a neutral CP-even Higgs boson at LEP II, less severe [2].

A possible cosmological domain wall problem [3] can be avoided by introducing suitable non-renormalizable operators [4] that do not generate dangerously large singlet tadpole diagrams [12].

Hence, the phenomenology of the NMSSM deserves to be studied at least as fully and precisely as that of the MSSM. Its particle content differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. Thus, the physics of the Higgs bosons – masses, couplings and branching ratios [1] and [5]–[11] can differ significantly from the MSSM. The purpose of the Fortran code NMHDECAY (Non Minimal Higgs Decays), that accompanies the present paper, is an accurate computation of these properties of the Higgs bosons in the NMSSM in terms of the parameters in the lagrangian. As its name suggests, the Fortran code uses to some extent — for MSSM-like processes — parts of the code HDECAY that is applicable to the Higgs sector of the MSSM [13].

In the present paper we define the NMSSM in terms of its parameters at the SUSY breaking scale. No assumption on the soft terms (like universal soft terms at a GUT scale) are made. The parameters in the Higgs sector are chosen as follows:

(*a*) Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda\, \widehat{S}\widehat{H}_u\widehat{H}_d + \frac{\kappa}{3}\, \widehat{S}^3 \tag{1.1}$$

depending on two dimensionless couplings λ, κ beyond the MSSM. (Hatted capital letters denote superfields, and unhatted capital letters will denote their scalar components).

(*b*) The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \,. \tag{1.2}$$

(*c*) The final two input parameters are

$$\tan\beta = \frac{\langle H_u\rangle}{\langle H_d\rangle}\ \mu_{\rm eff} = \lambda\,\langle S\rangle\,. \tag{1.3}$$

These, along with M_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S through the three minimization equations of the scalar potential.

Thus, as compared to two independent parameters in the Higgs sector of the MSSM (often chosen as $\tan\beta$ and M_A), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda\,, \qquad \kappa\,, \qquad A_\lambda\,, \qquad A_\kappa\,, \qquad \tan\beta\,, \qquad \mu_{\rm eff}\,. \tag{1.4}$$

We will choose sign conventions for the fields such that λ and $\tan\beta$ are positive, while κ, A_λ, A_κ and $\mu_{\rm eff}$ should be allowed to have either sign. For any choice of these parameters — as well as of the values for the gaugino masses and of the soft terms related to the squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths — NMHDECAY performs the following tasks:

1. It computes the masses and couplings of all physical states in the Higgs, chargino and neutralino sectors. Error messages are produced if a Higgs or squark mass squared is negative.

2. It computes the branching ratios into two particle final states (including charginos and neutralinos — decays to squarks and sleptons will be implemented in a later release) of all Higgs particles.

3. It checks whether the Higgs masses and couplings violate any bounds from negative Higgs searches at LEP, including many quite unconventional channels that are relevant for the NMSSM Higgs sector. It also checks the bound on the invisible Z width (possibly violated for light neutralinos). In addition, NMHDECAY checks the bounds on the lightest chargino and on neutralino pair production. Corresponding warnings are produced in case any of these phenomenological constraints are violated.

4. It checks whether the running Yukawa couplings encounter a Landau singularity below the GUT scale. A warning is produced if this happens.

5. Finally, NMHDECAY checks whether the physical minimum (with all vevs non-zero) of the scalar potential is deeper than the local unphysical minima with vanishing $\langle H_u \rangle$ or $\langle H_d \rangle$. If this is not the case, a warning is produced.

The web sites

`http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html`

`http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html`

provide links for downloading two different Fortran codes, along with descriptions, input files, and the data files that are required to apply the various LEP constraints.

The Fortran code NMHDECAY_SLHA.f reads from input files, and produces output files, that are as close as possible to the SUSY Les Houches Accord (SLHA) conventions [14]. Some generalizations of these conventions — including proposals for PDG numbers — have been necessary, however, in order to denote the NMSSM input parameters (1.4) and the additional particles in the Higgs and neutralinos sectors.

The Fortran code NMHDECAY_SCAN.f reads from "private" input files (samples are provided) that are constructed so as to scan over the NMSSM input parameters (1.4). Here the output is either "long" (easily human readable, if the number of points in parameter space is not too large), or "short", i.e. simple rows of numbers per point in parameter space, that should be edited according to the user's needs.

Note that the "long" output also gives the reduced couplings of all neutral Higgs bosons to gauge bosons (CV), up type quarks (CU), down type quarks (CD), two gluons (CG) and two photons (CGA) (all relative to a Standard Model Higgs boson of the same mass). Using these, it is easy to compute the NMSSM Higgs production cross sections at colliders by rescaling those for the SM Higgs boson.

The outline of the paper is as follows: in section 2, we describe in detail the accuracy with which the Higgs masses and mixing matrices are computed. In section 3, we describe the Higgs decay channels and corresponding accuracies that are used for the computation of the widths and branching ratios. In section 4, we describe the various phenomenological constraints that can be applied to the model. Section 5 contains a user's guide for the Fortran codes, as well as our generalizations of the SLHA conventions. In section 6, we show some results for branching ratios produced with the present code, emphasizing decay channels that are atypical of the MSSM, such as the lightest CP-even state h_1 decaying to 2 lighter CP-odd states $a_1 a_1$. Section 7 contains our conclusions. Appendix A contains a summary of our conventions, appendix B a summary of the Feynman rules, and appendix C the details of the radiative corrections to the higgs mass matrices.

2. Radiative corrections in the Higgs sector

Our convention for the superpotential, the soft terms, the resulting tree level potential and tree level mass matrices are given in appendix A. In the present section, we describe the accuracy with which radiative corrections are computed for the Higgs sector.

First, we assume that the Yukawa couplings and soft terms are defined at a SUSY breaking scale $Q = M_{\mathrm{SUSY}}$, which corresponds to an average of the squark masses. Quantum fluctuations at scales $> Q$ are assumed to be integrated out, which corresponds to the standard RG evolution of the parameters from a fundamental scale like M_{GUT} down to M_{SUSY}. The effective lagrangian at the scale Q can be assumed to be of the standard supersymmetric form plus soft terms.

We are interested in the full effective action

$$\Gamma_{\mathrm{eff}} = \sum_{\mathrm{i}} Z_i \; D_\mu H_i D^\mu H_i - V_{\mathrm{eff}}(H_i)\,, \tag{2.1}$$

that is obtained from the effective lagrangian at the scale Q by adding all quantum effects arising from fluctuations at scales $\lesssim Q$. (Here, H_i denotes all the Higgs fields, H_u, H_d and S.) These quantum effects can be classified according to powers of the various couplings, and powers of potentially large logarithms. Explicit formulas for the radiative corrections to V_{eff}, Z_i and hence to the Higgs mass matrices are given in appendix C, subsequently we describe only the orders to which radiative corrections are taken into account.

Let us start with the corrections to the effective potential. It is somewhat more convenient, however, to classify the corrections to the (lightest) scalar Higgs mass, which is essentially the second derivative of the effective potential. At tree level this mass squared is bounded by

$$m_1^2 < M_Z^2 \left(\cos^2 2\beta + \frac{2\lambda^2 \sin^2 2\beta}{g_1^2 + g_2^2} \right) , \tag{2.2}$$

which is of the order

$$m_1^2 \sim \mathcal{O}\left(g^2 v^2\right) + \mathcal{O}\left(\lambda^2 v^2\right) , \tag{2.3}$$

where g denotes the electroweak gauge couplings, and v the magnitude of the Higgs vacuum expectation values (we do not distinguish between large and small $\tan\beta$ in this section).

The dominant one-loop corrections to m_1^2 originate from top, stop, bottom and sbottom loops. The corresponding corrections δm_1^2 to m_1^2 are of order

$$\begin{aligned}
\delta_{\mathrm{top}} m_1^2 &\sim h_t^4 \; v^2 \ln\left(\frac{Q^2}{m_{\mathrm{top}}^2}\right) , \\
\delta_{\mathrm{stop}} m_1^2 &\sim h_t^4 \; v^2 \qquad (\text{for } Q^2 \sim m_{\mathrm{stop}}^2) , \\
\delta_{\mathrm{bottom}} m_1^2 &\sim h_b^4 \; v^2 \ln\left(\frac{Q^2}{m_{\mathrm{bottom}}^2}\right) , \\
\delta_{\mathrm{sbottom}} m_1^2 &\sim h_b^4 \; v^2 \qquad (\text{for } Q^2 \sim m_{\mathrm{sbottom}}^2) .
\end{aligned} \tag{2.4}$$

We have included these contributions [5, 6, 10] exactly, without expanding in large logarithms or squark mass splittings.

The dominant two loop corrections are of the form [10, 11]

$$\delta^{(2)} m_1^2 \sim h_t^6 v^2 \ln^2\left(\frac{Q^2}{m_{\mathrm{top}}^2}\right) + h_t^4 \alpha_s v^2 \ln^2\left(\frac{Q^2}{m_{\mathrm{top}}^2}\right) . \tag{2.5}$$

Here, we only include the leading double logarithms, i.e. we neglect single logs as well as terms involving h_b^2.

Coming back to the one-loop order, we focus on including all contributions of the order $g^4\times$ large logarithms. These can be obtained by RG methods [15], taking care of the fact that the electroweak couplings g are defined at the scale M_Z and not at the scale M_{SUSY}. We have included all these contributions as given in [15], distinguishing large logarithms $\ln(Q^2/M_Z^2)$ from $\ln(M_A^2/M_Z^2)$ and $\ln(\mu_i^2/M_Z^2)$ ($\mu_i = M_1, M_2$ or μ_{eff}, where M_i are the electroweak gaugino masses). We have not (yet) included terms of the order $g^2\lambda^2\times$ large logarithms, nor terms of the order $g^4\times$ finite.

This concludes our contributions to the effective potential. Next, we outline the contributions to the wave function normalization constants Z_i in (2.1). If evaluated for external momenta of $\mathcal{O}(m_{\mathrm{top}})$ (the order of the Higgs masses), top and bottom quark loops yield contributions to δZ_i (relative to 1) of the form

$$\delta Z_1 \sim h_t^2 \; \ln\left(\frac{Q^2}{m_{\mathrm{top}}^2}\right) , \qquad \delta Z_2 \sim h_b^2 \; \ln\left(\frac{Q^2}{m_{\mathrm{top}}^2}\right) . \tag{2.6}$$

After rescaling the Higgs fields so that their kinetic energies are canonically normalized, these effects generate contributions $\delta_Z m_1^2$ which take the form

$$\delta_Z m_1^2 \sim g^2 h_{t/b}^2 v^2 \ln\left(\frac{Q^2}{m_{\rm top}^2}\right) + \lambda^2 h_{t/b}^2 v^2 \ln\left(\frac{Q^2}{m_{\rm top}^2}\right). \tag{2.7}$$

However, after diagonalization of the Higgs mass matrices, one may find eigenstates with masses m_H substantially larger than $m_{\rm top}$. Then the Z factors should be evaluated at external momenta of $\mathcal{O}(m_H)$, i.e. at the pole of the propagators. Hence for $m_H \gg m_{\rm top}$ the logarithms $\ln(Q^2/m_{\rm top}^2)$ in (2.6) should be replaced by $\ln(Q^2/m_H^2)$, with coefficients depending on Higgs mixing matrices.

All this is done in the program, with the net result that all potentially large logarithms with coefficients $g^2 h_{t/b}^2$ or $\lambda^2 h_{t/b}^2$ have the correct arguments. (We neglect, however, contributions of $\mathcal{O}(\ln(m_H^2/m_{\rm top}^2))$ in the off diagonal Higgs matrix elements, which affects only the couplings of very heavy Higgses.) Since we have neglected terms of $\mathcal{O}(h_{t/b}^2 \times finite)$ in the Z factors and electroweak corrections without large logarithms, the dominant sources of uncertainty in δm^2 are contributions of $\mathcal{O}(g^4 v^2 \times finite)$, $\mathcal{O}(g^2 h_{t/b}^2 v^2 \times finite)$ and $\mathcal{O}(\lambda^2 h_{t/b}^2 v^2 \times finite)$ as well as subdominant (single log) two loop corrections.

We have checked that in the end, after expanding the Z factors appropriately, all Q^2 dependence in the one-loop contributions can be reabsorbed into the running of the input parameters that are defined at the scale Q^2. An exception is the Q^2 dependence of terms $\sim g^4$, since we have not included the squark loop contributions $\sim g^4$ (from squark couplings to Higgs bosons arising from the D-terms). These would give contributions to m_1^2 of the order

$$\delta_D m_1^2 \sim g^4 \ln\left(\frac{Q^2}{m_{\rm squark}^2}\right), \tag{2.8}$$

which is relatively small for $Q^2 \sim m_{\rm squark}^2$.

Since we have not included contributions of the orders $(g^4,\ g^2 h_{t,b}^2)\times$ finite, we are actually not sensitive to the scheme in which the electroweak couplings are defined. Since in the computation of the decay widths we use the physical masses M_W and M_Z in order to parameterize some couplings, the most useful scheme here is the on-shell scheme, where the three "unphysical" electroweak parameters $v^2 = \langle H_u^2 + H_d^2\rangle$, g_1^2 and g_2^2 are defined in terms of G_F, M_W and M_Z as

$$\begin{aligned} v^2 &= \frac{1}{2\sqrt{2}\, G_F}, \\ g_2^2 &= 2\frac{M_W^2}{v^2}, \\ g_1^2 + g_2^2 &= 2\frac{M_Z^2}{v^2}. \end{aligned} \tag{2.9}$$

Finally we remark that we have applied the above corrections consistently to the complete Higgs sector, i.e. the CP-even, CP-odd and charged Higgs states. One finds that one can reabsorb several of the radiative corrections into a redefinition of the input parameter A_λ [5], after which the CP-odd mass matrix assumes its tree level form, up to the required rescalings of the vevs by the Z factors. This phenomenon is familiar from the MSSM, where M_A is thus a convenient input parameter, and the required rescalings are often absorbed into a scale dependent value of $\tan\beta$.

We have checked that in the MSSM limit of the NMSSM ($\lambda, \kappa \ll 1$), after comparing for the same values of $\tan\beta$ and M_A (and the remaining soft terms), the mass of the lightest CP-even state agrees with the one computed in HDECAY [13] (models 1 [16] or 2 [17]) to better than 2 GeV for moderate mixing ($A_{\rm top} \lesssim 1$ TeV), and to better than 3 GeV for maximal mixing. This coincides with the expected theoretical error on this mass given the orders of uncertainty discussed above.

A more detailed discussion of the radiative corrections to the NMSSM Higgs sector will appear in [18].

3. Higgs decays

In this section we describe the decay modes that are included in the Fortran code. Most of the corresponding code is extracted from HDECAY [13]. In what follows, H denotes any of the 3 CP-even or 2 CP-odd scalars, or the charged Higgs boson.

(*a*) $H \to$ gluons: We take into account charm, bottom and top quark loops; the lowest order contribution to the decay width is given in [19]. QCD radiative corrections [20] are included at the same level as in HDECAY. Squark loops are not (yet) taken into account.

(*b*) $H \to$ leptons and quarks: QCD corrections are included as in HDECAY [21], but we use always the two loop formula for the relation between the pole quark mass $m_{\rm pole}$ and the running $\overline{MS}$ quark mass $\overline{m}(m_{\rm pole})$:

$$\overline{m}_i(m_{\rm pole}) = \frac{m_{\rm pole}}{1 + \frac{4\alpha_s(m_{\rm pole})}{3\pi} + K_i \left(\frac{\alpha_s(m_{\rm pole})}{\pi}\right)^2} \tag{3.1}$$

with $K_{\rm bottom} \sim 12.4$, $K_{\rm top} \sim 11$. The scale dependence of the running s and c quark masses is computed as in HDECAY, but for the running b and t quark masses we use running Yukawa couplings that solve the one-loop RG equations exactly in α_s, improved by corrections to first order in the Yukawa couplings themselves (see appendix C).

Since electroweak corrections to decays into leptons and quarks are small [13], they are neglected. Below-threshold three-body decays $H \to t\bar{t} \to t\bar{b}W^-$ are not (yet) taken into account. However, contributions to the decay rates into $b\bar{b}$ ($c\bar{c}$) from $H \to gg^* \to gb\bar{b}$ ($c\bar{c}$) are included as in HDECAY (option NF-GG = 3, that can easily be modified by re-editing the line NFGG = 3 at the beginning of the subroutine DECAY). Susy loop corrections are not (yet) included. (These could be important for light squarks and/or large $\tan\beta$.) In the case of a decaying charged Higgs boson, we must input the CKM matrix elements V_{us}, V_{ub} and V_{cb}.

(c) $H \to WW/ZZ$: We treat these decays as in HDECAY [22], but without the double off-shell option.

(d) $H \to \gamma\gamma/Z\gamma$: We take into account loops of charm, bottom and top quarks, τ leptons, W bosons, charginos and charged Higgs bosons [23]. Loops of squarks and sleptons are not (yet) considered.

(e) $H \to HH$: All kinematically possible combinations (H = CP-even, CP-odd or charged) are considered, with (Higgs)^3-couplings computed from the NMSSM potential (see F. Franke and H. Fraas in ref. [1] and appendix B). The dominant (leading log) radiative corrections from top-quark and bottom-quark loops to these couplings are included in the Fortran code, the corresponding formulas will appear in [18]. Decays into three particle final states are not (yet) taken into account.

(f) $H \to HZ/HW$: All kinematically possible combinations (H = CP-even, CP-odd or charged) are considered [24]. Below threshold decays into three particle final states are not (yet) taken into account.

(g) $H \to$ charginos/neutralinos: First, the 2×2 chargino and 5×5 neutralino mass matrices are diagonalized in the subroutines CHARGINO and NEUTRALINO, and the decays into all possible two body final states [25] are included in the program.

Decays into sleptons or squarks are not (yet) considered. Note that relatively light squarks — even if too heavy to be produced in Higgs decays — could nevertheless affect notably the processes b) and d) above through squark loops. For this reason the present version of NMHDECAY should not be applied to scenarios with light squarks.

4. Constraints on the parameter space

Once the spectrum and decay widths of the Higgses are computed, we check some experimental and theoretical constraints for each point in parameter space:

First, we require that there is no Landau pole for the Yukawa couplings $\lambda, \kappa, h_{\mathrm{top}}$ and h_{bot} below the GUT scale. For this, we integrate the two loop renormalization group equations for the Yukawa and gauge couplings from the Susy scale up to the point where the gauge couplings g_1 and g_2 unify. At this point we check that none of the couplings is too large (i.e. exceeds $\sqrt{4\pi}$). (For simplicity we assume a universal Susy threshold at $M_{\mathrm{susy}} \sim Q$.)

We also check that there is no deeper minimum of the scalar potential with vanishing $\langle H_u \rangle$ or $\langle H_d \rangle$.

Finally we check all available experimental constraints from LEP:

1. In the neutralino sector, we check that the lightest neutralino does not contribute too much to the invisible Z width ($\Gamma(Z \to \chi_1^0\chi_1^0) < 1.76\,\mathrm{MeV}$ [26]) if $m_{\chi_1^0} < M_Z/2$, and that $\sigma(e^+e^- \to \chi_1^0\chi_i^0) < 10^{-2}$ pb if $m_{\chi_1^0} + m_{\chi_i^0} < 209\,\mathrm{GeV}$ ($i > 1$) and $\sigma(e^+e^- \to \chi_i^0\chi_j^0) < 10^{-1}$ pb if $m_{\chi_i^0} + m_{\chi_j^0} < 209\,\mathrm{GeV}$ ($i, j > 1$) [27]. In the chargino sector, we check that the lightest chargino is not too light ($m_{\chi_1^+} < 103.5\,\mathrm{GeV}$ [28]).

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2. In the charged Higgs sector, we check the bound $m_{H^+} > 78.6\,\mathrm{GeV}$ [29].

3. In the neutral Higgs sector, we check the constraints on the production rates (reduced couplings) $\times$ branching ratios versus the masses, for all of the CP-even states h and CP-odd states a, in the following channels studied at LEP:

   - $e^+e^- \to hZ$, with $h \to b\bar{b}$ and $h \to \tau^+\tau^-$ (from the LEP Higgs working group results [30]);
   - $e^+e^- \to hZ$, with h decaying to two jets. For this, we combined the low mass range results from OPAL at LEP2 [31] with the higher mass range obtained at LEP2 by the LEP Higgs Working group [32];
   - $e^+e^- \to hZ$, with $h \to \gamma\gamma$ (from the LEP Higgs working group results [33]);
   - $e^+e^- \to hZ$, with h decaying invisibly (i.e. into two neutralinos). For this, we combined the low mass range results from ALEPH at LEP1 [34] with the higher mass range obtained at LEP2 by the LEP Higgs Working group [35].
   - $e^+e^- \to hZ$, independent of the h decay mode, looking for a peak of the M_X recoil mass distribution in $e^+e^- \to XZ$. For this, we combined the low mass range results from ALEPH at LEP1 [34] with the higher mass range obtained at LEP2 by OPAL [36].
   - For the associated production mode $e^+e^- \to ha$ we used the DELPHI results [37] for the various final states: $ha \to 4b$, $ha \to 4\tau$ and $ha \to aaa \to 6b$. The latter decay channel plays an important role in the NMSSM, where the lightest CP-even Higgs can decay mainly into two light CP-odd Higgses over large areas of the parameter space, as we will see in section 6. (In the context of the CP-conserving MSSM, this is possible only for very special parameter choices.) DELPHI also studied the channel $hZ \to aaZ \to 4b\ +\ 2\mathrm{jets}$ [37]; their limits have also been implemented in our phenomenological constraints.
   - The channels $e^+e^- \to hZ \to aaZ$ with $aa \to 4\mathrm{jets}$, $2\mathrm{jets} + cc$, $2\mathrm{jets} + \tau\tau$, 4τ's, $cccc$, $\tau\tau + cc$ have been studied by OPAL [38] (see their figure 7 — we employ numerical files for the contours provided by the authors). These limits have also been included.

All these constraints are contained in the files located in the directory LEPCON downloadable together with the Fortran codes. The file names are constructed so that they can be identified with the above delineated constraints. (The current numerical limits contained in these files can possibly be improved with the help of further dedicated analysis of existing data.)

5. How to use NMHDECAY

NMHDECAY exists in two versions:

1. NMHDECAY_SLHA uses an input file and produces output files that are suitable generalizations of the SLHA conventions [14]. It is designed for studying the properties of one user-defined point in parameter space.

2. NMHDECAY_SCAN uses input and output files that can be modified by the user so as to scan over parts of, or all of the NMSSM parameters defined in eq. (1.4).

Both programs have the common feature that they are based on one single Fortran code (NMHDECAY_SLHA.f or NMHDECAY_SCAN.f) that does not need to be linked with any other code. However, both programs need data files in order to check against negative Higgs searches at LEP in the numerous channels discussed in section 4. These data files are available in the directory LEPCON, that can be downloaded from the NMHDECAY home page. Note that the directory LEPCON has to be situated in the same directory that contains the executable NMHDECAY code. We now discuss the particular features of the two programs.

5.1 NMHDECAY_SLHA

NMHDECAY_SLHA uses an input file `slhainp.dat`, a version of which is downloaded automatically with the Fortran code. This sample file appears in Table 1. Several comments on its contents are in order.

(*a*) "BLOCK MODSEL" contains the entry 3 (corresponding to the choice of the particle content) with switch 1, as attributed to the NMSSM in [14].

(*b*) "BLOCK SMINPUTS" contains important Standard Model parameters.

  (a) First, there is the inverse electromagnetic coupling constant at the scale 0, which is that required for the computations of the decay widths into two (on-shell) photons.

  (b) Second, various Higgs couplings are defined in terms of M_Z, $\alpha_s(M_Z)$, M_W and G_F, which means an on shell scheme is implicitly being used in order to define the electroweak parameters (cf. the corresponding discussion in section 2).
  M_Z, $\alpha_s(M_Z)$ and G_F are input as part of "BLOCK SMINPUTS", whereas the numerical value of $M_W = 80.42\,\mathrm{GeV}$ is defined in the subroutine INPUT, that reads also the input file `slhainp.dat`. (Part of the corresponding code is copied from `pyslha.f`, with thanks to P. Skands.)

  (c) Third, as part of this block we input the running b quark mass $m_b(m_b)$, the top quark pole mass and m_τ.

  (d) In addition, NMHDECAY needs the (pole) quark masses m_s and m_c, as well as the CKM matrix elements V_{us}, V_{cb} and V_{ub}. The numerical values of these five parameters are defined in the subroutine INPUT. (For convenience, they are printed out in the output file `spectr.dat`, see below.)

c) "BLOCK MINPAR" contains only the switch 3 with the input value for $\tan\beta$ specified.

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(*d*) "BLOCK EXTPAR" contains inputs for the SUSY and soft-SUSY-breaking parameters.

Needed generalizations of the SLHA conventions appear. The new entries are:

61	for λ
62	for κ
63	for A_λ
64	for A_κ

Note that the entry 23 (μ) is used for the effective μ parameter, $\mu_{\text{eff}} = \lambda \langle S \rangle$, in the NMSSM. It should also be noted that neither the slepton entries (all generations) nor the squark entries for the first and second generations are actually used; these are included for future use only.

The two output files of NMHDECAY_SLHA are `spectr.dat` (see table 2) and `decay.dat`. In `spectr.dat`, "BLOCK SPINFO" is followed by warnings (switch 3) if any of the constraints described in section 4 are violated, or if any of the six Higgs states could decay into squarks in which case the corresponding widths and branching ratios should not be used. This segment of the output also displays error messages (switch 4) if any of the Higgs, stop or sbottom states have a negative mass squared. No spectrum output is produced in this case.

In the output, "BLOCK SMINPUTS" is followed by a printout of the Standard Model input parameters. The numerical values for M_W, m_s, m_c, V_{us}, V_{cb} and V_{ub}, that have no SLHA numbers, appear in lines subsequent to "# SMINPUTS Beyond SLHA". Similarly, "BLOCK MINPAR" is followed by a printout of the value of $\tan\beta$ and "BLOCK EXTPAR" is followed by a printout of the important SUSY and soft-SUSY-breaking parameters.

The (pole) masses for the Higgs particles, the neutralinos, the charginos and squarks appear in the output following "BLOCK MASS". There, one finds several essential NMSSM generalizations of the SLHA conventions. The new entries, with proposed PDG codes, are

45	for the third CP-even Higgs boson,
46	for the second CP-odd Higgs boson,
1000045	for the fifth neutralino.

The Higgs mixings in the CP-even sector follow "BLOCK HIGMIX" and those in the CP-odd sector follow "BLOCK AMIX". Both segments of the output contain NMSSM generalizations of the SLHA conventions, that are required in order to parameterize the mixing in the enlarged Higgs sector. The meaning of the matrix elements S_{ij} (i, j = 1, 2, 3) and P'_{ij} (i, j = 1, 2) is as follows.

- According to the SLHA conventions the Higgs states H_u, H_d are denoted by H_2, H_1, respectively. (Inside the Fortran code the Higgs states H_u, H_d are denoted by H_1, H_2, which is of no relevance for the SLHA output.) Hence, for the purpose of the SLHA output, the CP-even Higgs weak eigenstates are numbered by $S_i^{weak} = (H_{dR}, H_{uR}, S_R)$ (R refers to the real component of the field). If h_i are the mass eigenstates (ordered in mass), the convention is $h_i = S_{ij} S_j^{weak}$.

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- In the CP-odd sector the bare Higgs fields are H_{uI}, H_{dI}, S_I (I for imaginary component). Again, for the purpose of the SLHA output, the CP-odd Higgs weak eigenstates are denoted by $H_{uI} = H_{2I}$, $H_{dI} = H_{1I}$. The mass eigenstates are a_i (ordered in mass, $i = 1, 2$) and the Goldstone mode $\tilde{G}$. Then the elements of P'_{ij} are defined as

$$
\begin{aligned}
H_{1I} &= \sin\beta(P'_{11}a_1 + P'_{21}a_2) + \cos\beta\tilde{G}\,, \\
H_{2I} &= \cos\beta(P'_{11}a_1 + P'_{21}a_2) - \sin\beta\tilde{G}\,, \\
S_I &= P'_{12}a_1 + P'_{22}a_2\,.
\end{aligned} \tag{5.1}
$$

- In the output, "BLOCK NMIX" is followed by a printout of the obvious generalization of the 4×4 MSSM neutralino mixing matrix to the 5×5 NMSSM neutralino mixing matrix (with real entries); "BLOCK UMIX" and "BLOCK VMIX" are followed by printouts of the U and V matrices as defined in the MSSM.

- The output file `decay.dat` respects the SLHA conventions for the Decay file (at present we consider two particle final states only), using the above generalizations of the PDG codes both for the decaying particle and the final states.

5.2 NMHDECAY_SCAN

NMHDECAY_SCAN uses the input files `smpar.dat` and `scaninp.dat`, versions of which are downloaded automatically with the Fortran code. One of the sample files appears in table 3.

In the file `smpar.dat` the following standard model parameters must be specified: $\alpha_s(M_Z)$, G_F, $\alpha^{-1}_{\text{e.m.}}(0)$, the lepton masses m_τ and m_μ, M_Z, M_W, the pole quark masses m_s, m_c, the running bottom quark mass $m_b(m_b)$, the top quark pole mass m_t, and the CKM matrix elements V_{us}, V_{cb} and V_{ub}.

In the file `scaninp.dat`, the following must be specified:

- the total number of points to be scanned in parameter space;

- the output format (0 for "short", corresponding to simple rows of numbers per allowed point in parameter space, and 1 for "long", as described below);

- lower and upper limits for the NMSSM parameters λ, κ, $\tan\beta$, μ_{eff}, A_λ and A_κ;

- the soft squark masses, trilinear couplings and gaugino masses over which no scan is performed — the slepton masses as well as the squark masses of the first two generations (without index 3) are not (yet) used.

The scan in parameter space uses a random number generator, such that all NMSSM parameters are randomly chosen point by point in the parameter space within the specified limits.

The output file containing the physical parameters is always called `scanout.dat`, regardless of the output format chosen. The numbers printed out for the output format 0 (recommended for scans over more than 10 points in parameter space) should be edited according to the user's needs.[1] The output format 1 is easily readable and shows

- the NMSSM parameters for each point,
- possible warnings in case any phenomenological constraint is violated, a strong coupling regime below $M_{\rm GUT}$ appears or an unphysical minimum is deeper than the physical one (cf. section 4), if any of the six Higgs states could decay into squarks (in which case the corresponding widths and branching ratios should not be used) or error messages ("fatal" errors) in case any of the Higgs, stop or sbottom states has a negative mass squared (in which case no additional output is produced).
- for each of the six Higgs states, their mass, their decomposition into weak eigenstates (H_u, H_d, S) (in `Components`), their reduced couplings to gauge bosons (CV), up type quarks (CU), down type quarks (CD), two gluons (CG) and two photons (CGA) (all relative to a standard model Higgs boson with the same mass), their branching ratios (where "Higgses" denote all possible two Higgs final states, and "sparticles" all possible two particle neutralino/chargino final states), and their total width,
- the neutralino and chargino masses (all masses in GeV), as well as the neutralino composition in the basis $\psi^0 = (-i\lambda_1, -i\lambda_2, \psi_u^0, \psi_d^0, \psi_s)$ described in detail in appendix A.
- the (pole) stop and sbottom masses (in GeV).

The output file `scanerr.dat` shows how many of the points in parameter space have avoided fatal errors or violations of phenomenological constraints, and the range in the NMSSM parameter space over which points have passed all these tests.

For users who wish to call a subroutine as a function of the Higgs, chargino, neutralino and squark sector outputs, including mixing angles and other parameters and quantities computed during the course of the scan, they should use the parameters and common blocks found in NMHDECAY_SCAN.f within "SUBROUTINE OUTPUT". The comments included in this subroutine should allow easy identification of all the parameters, branching ratios, mixing angles and so forth that would be of potential interest for inputting into a user's subroutine.

6. Results and discussion

As stated above, the masses, couplings and decay properties of the Higgs bosons of the NMSSM can differ significantly from the MSSM. The primary purpose of NMHDECAY is to allow for detailed studies of such cases.

A particularly interesting case is where the lightest Higgs scalar h_1, although primarily non-singlet, decays mainly into two pseudoscalars (also primarily non-singlet) [8]. It is then interesting to see how the branching ratios of h_1 depend on the parameters of the model. In particular, we can determine whether the choices of parameters that yield the above types

[1] See the section of the program following the comment line " The following 3 lines should be edited according to the user's needs".

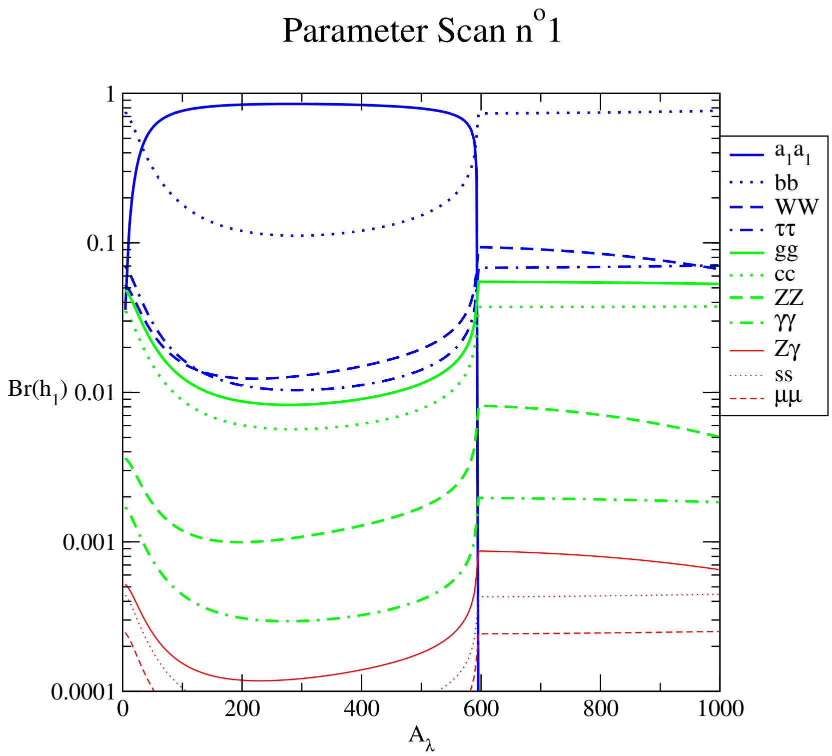

Figure 1: Branching ratios of h_1 as a function of A_λ for $\lambda = \kappa = 0.3$, $\tan\beta = 5$, $\mu_{\rm eff} = 180\,\rm GeV$, $A_\kappa = 0$, $m_{\rm squark} = 1\,\rm TeV$, and $A_t = 1.5\,\rm TeV$.

of decays are ruled out for some of the phenomenological reasons discussed in section 4. We can also determine if there are particular (fine-tuned) relations between the parameters required for $h_1 \to a_1 a_1$ to be dominant. To study such issues, it will be convenient to fix all but one of the parameters (which allows for a reasonable graphical representation), and perform a scan over the remaining parameter.

In some sense, the input parameter A_λ is the most natural one to vary, since the mass of the MSSM like pseudoscalar depends quite strongly on A_λ (and hence A_λ plays the role of M_A in the MSSM).

Let us first consider the following choice of the NMSSM parameters (cf. eq. (1.4)): $\lambda = \kappa = 0.3$, $\tan\beta = 5$, $\mu_{\rm eff} = 180\,\rm GeV$, $A_\kappa = 0$. For the squark masses and trilinear couplings, we take $1\,\rm TeV$ and $1.5\,\rm TeV$, respectively. Varying A_λ between 0 and $1000\,\rm GeV$, we obtain the branching ratios for h_1 as shown in figure 1. These show clearly that, for $A_\lambda \lesssim 600\,\rm GeV$, the decay $h_1 \to a_1\, a_1$ is dominant.

The reason for the sharp drop of the $h_1 \to a_1 a_1$ branching ratio for $A_\lambda > 600\,\rm GeV$ becomes clear from figure 2, where we show the masses m_{h_1} and m_{a_1} as functions of A_λ; for $A_\lambda > 600\,\rm GeV$, m_{a_1} becomes larger than $m_{h_1}/2$. The $h_1 \to a_1 a_1$ decay is also reduced as $A_\lambda \to 0$, even though the a_1 becomes very light, because in this case it is mainly singlet and has a very small coupling to the h_1. None of the points in these two graphs are excluded by LEP. They should be visible at the LHC using the techniques we have developed [8] for isolating the $WW \to h \to aa$ type of signal.

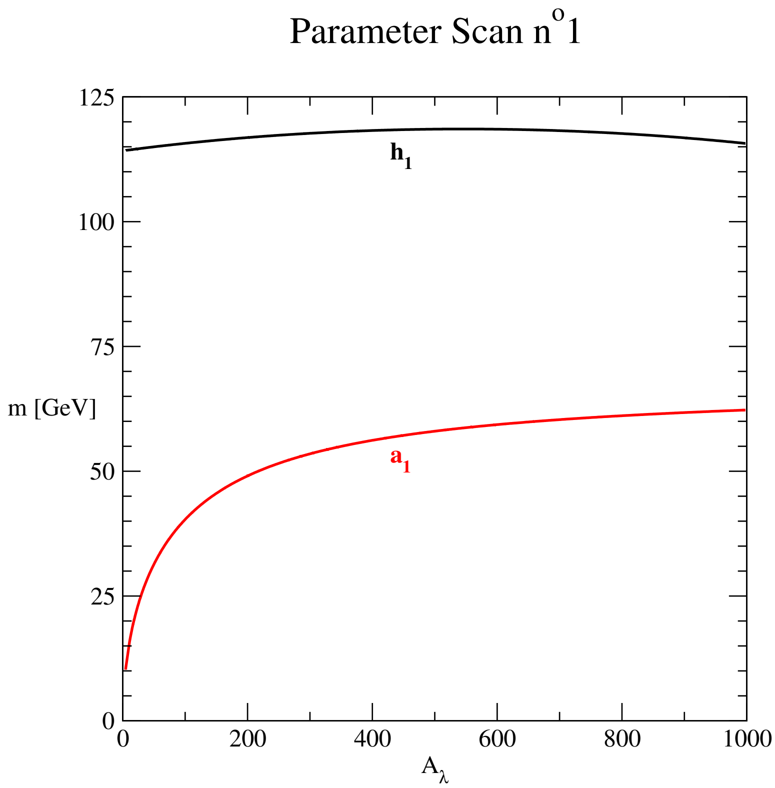

Figure 2: m_{h_1} and m_{a_1} as a function of A_λ for the same parameters as in figure 1.

For a second sample set of plots, figures 3–6, we take $\lambda = 0.5$, $\kappa = -0.15$, $\tan\beta = 3.5$, $\mu_{\rm eff} = 200\,{\rm GeV}$, $A_\lambda = 780\,{\rm GeV}$ and $A_\kappa \in [150\,{\rm GeV}, 250\,{\rm GeV}]$. The `scaninp.dat` file for this case is given in table 3. For much of this parameter range, neither the h_1 nor the h_2 would have been observable at LEP. In particular, figure 5 shows that $m_{h_2} \gtrsim 120\,{\rm GeV}$ implying that the h_2 is beyond the LEP kinematical reach. The h_1 is much lighter. However, this light Higgs is not excluded by LEP over most of the above A_κ range since: a) its reduced coupling to gauge bosons is small; and b) $h_1 \to b\bar{b}$ is suppressed so that $h_1 \to jj$ decays are dominant (see figure 3). In figure 6, we plot $\xi^2 = C_V(h_1)^2 \times BR(h_1 \to jj)$ for our selected points as well as the region excluded by LEP searches in this channel [32]. We see that only if $m_{h_1} \lesssim 53\,{\rm GeV}$, which corresponds to $A_\kappa \gtrsim 235\,{\rm GeV}$, would the h_1 be excluded by LEP data.

Will these Higgs bosons be observable at the LHC? In this regard, it is important to note from figure 4 that when $A_\kappa \gtrsim 215\,{\rm GeV}$, $h_2 \to h_1 h_1$ decays are dominant. This occurs because m_{h_1} decreases with A_κ, see figure 5. Meanwhile, figure 3 shows that $BR(h_1 \to b\bar{b})$ and $BR(h_1 \to \tau^+\tau^-)$ are both small when $A_\kappa \in [205\,{\rm GeV}, 220\,{\rm GeV}]$; in this region of parameter space, the h_1 decays mainly to $c\bar{c}$ or gg. Thus, for $A_\kappa \sim 215 - 220\,{\rm GeV}$:

- The h_1 has a mass that lies below the mass range currently studied for Higgs detection at the LHC. Further, the h_1 will be so weakly produced at the LHC (since $\xi^2 \lesssim 0.1$) that extensions to lower Higgs masses of the current LHC studies would probably conclude it was undetectable.

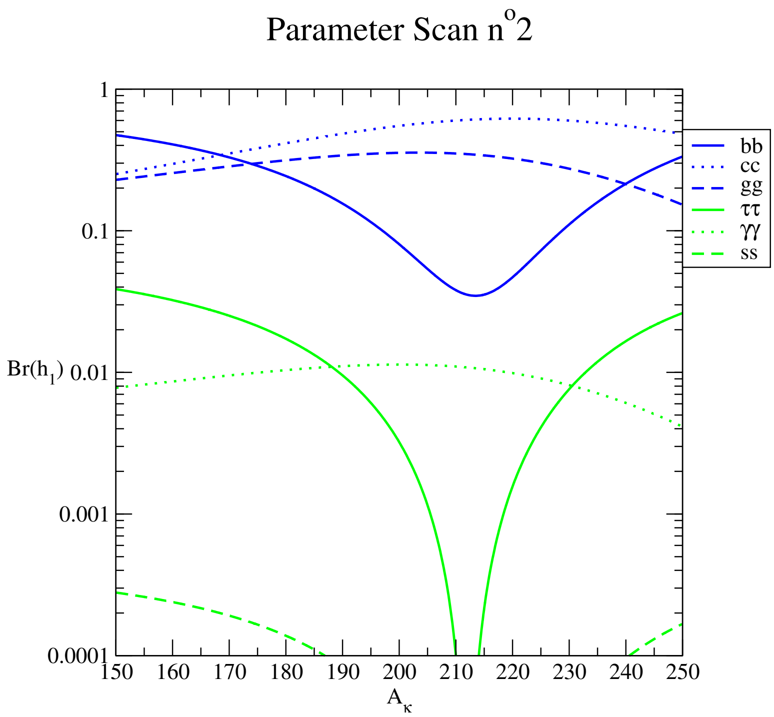

Figure 3: Branching ratios of h_1 as a function of A_κ for $\lambda = 0.5$, $\kappa = -0.15$, $\tan\beta = 3.5$, $\mu_{\rm eff} = 200\,\text{GeV}$, $A_\lambda = 780\,\text{GeV}$, $m_{\rm squark} = 1\,\text{TeV}$, and $A_t = 1.5\,\text{TeV}$.

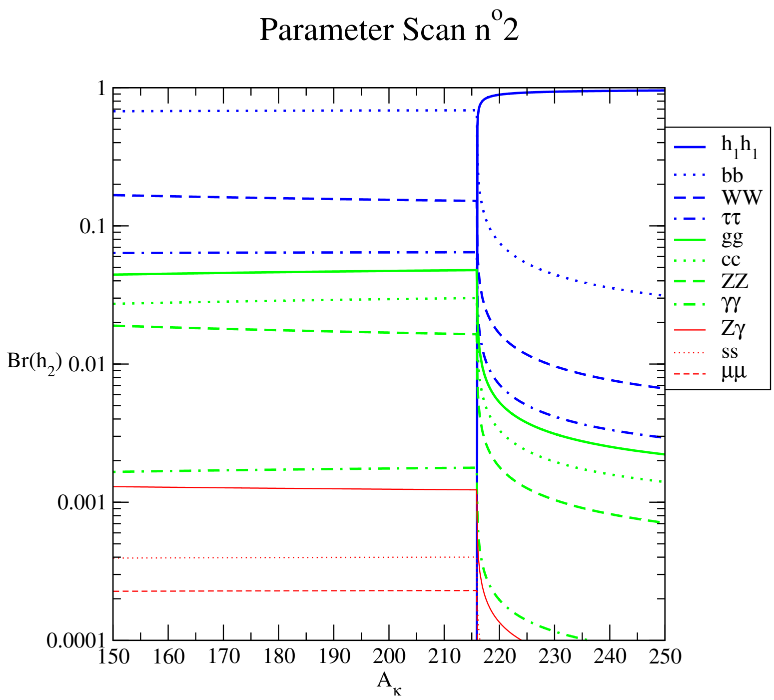

Figure 4: Branching ratios of h_2 as a function of A_κ for the same parameter choices as in figure 3.

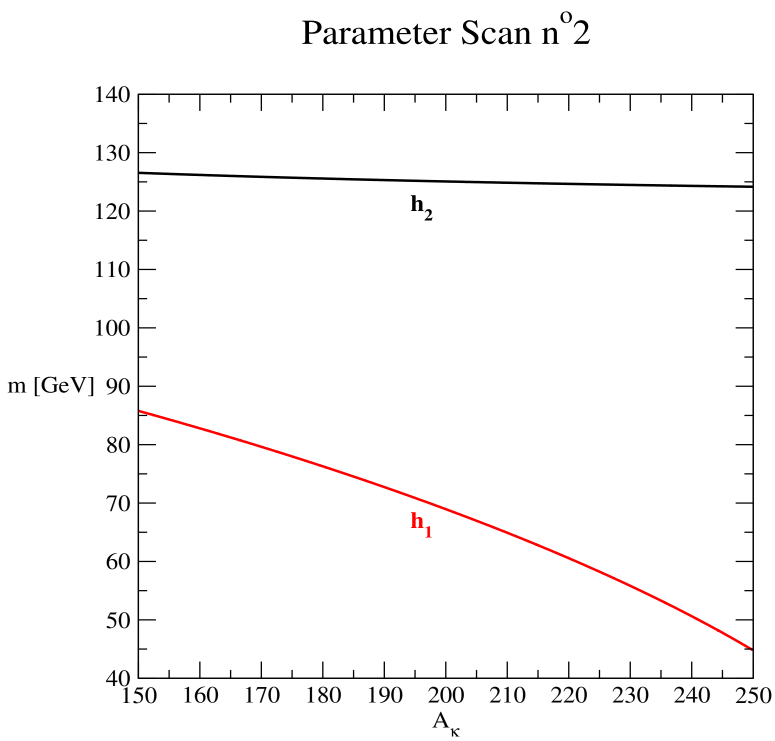

Figure 5: m_{h_1} and m_{h_1} as a function of A_κ for the same parameters as in figure 3.

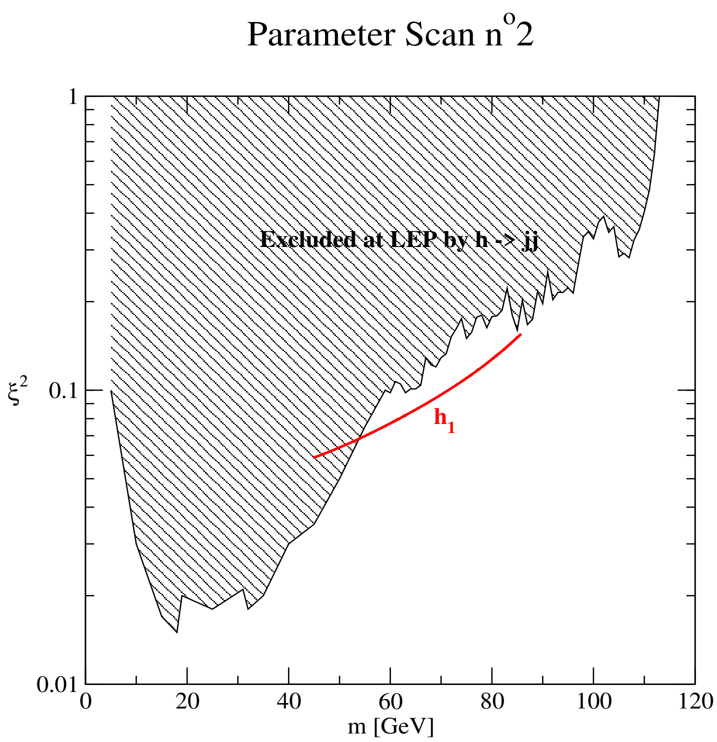

Figure 6: LEP constraints in comparison to predictions for h_1 for the parameters of figure 3. Note the correlation of $m = m_{h_1}$ with A_κ given in figure 5.

- Simultaneously, the strongly produced h_2 has decays dominated by $h_2 \to h_1 h_1$ with $h_1 \to c\bar{c}, gg$ (but not $b\bar{b}$ or $\tau^+\tau^-$). As a result, the techniques of [8] for $h \to aa$ (which require a significant $a \to \tau^+\tau^-$ branching ratio) do not apply, and the h_2 would also appear to be very difficult to observe at the LHC.

We note that the above choice of parameters produces a phenomenology for the Higgses that is somewhat similar to that discussed in [9], which focuses on a region of parameter space such that the h_1 has suppressed decays to $b\bar{b}$ and $\tau^+\tau^-$. The difference is that the decay of their h_2 to $h_1 h_1$ is kinematically forbidden and the h_2 could be detected at the LHC.

7. Conclusions

In this paper, we have presented details regarding the now publicly available NMHDECAY programs, NMHDECAY_SLHA.f and NMHDECAY_SCAN.f, that can be used to explore the Higgs sector of the Next to Minimal Supersymmetric Model defined by adding one singlet Higgs superfield to the Minimal Supersymmetric Model. The programs, and associated data files, can be downloaded from the two web pages:

`http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html`

`http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html`

The web pages provide simplified descriptions of the programs and instructions on how to use them. The programs will be updated to include additional features and refinements in subsequent versions. We welcome comments with regard to improvements that users would find helpful.

We have exemplified the use of these programs for two particularly interesting scenarios: The first illustrates the potentially crucial importance of the LHC $h \to aa$ detection mode (that is dominant over a significant, not fine-tuned, range of parameters of the NMSSM). The second exposes a limited portion of parameter space for which Higgs discovery would not have occurred at LEP and will probably not be possible at the LHC.

Acknowledgments

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```
#  INPUT FILE FOR NMHDECAY
#  BASED ON SUSY LES HOUCHES ACCORD, MODIFIED FOR THE NMSSM
#  IN EXTPAR: LINES 61-64: NMSSM YUKAWA COUPLINGS AND TRILIN. SOFT TERMS
BLOCK MODSEL
  3     1     # NMSSM PARTICLE CONTENT
BLOCK SMINPUTS
  1    137.036 # ALPHA_EM^-1(0)
  2    1.16639D-5   # GF
  3    0.12    # ALPHA_S(MZ)
  4    91.187  # MZ
  5      4.24  # MB(MB), RUNNING B QUARK MASS
  6    175.    # TOP QUARK POLE MASS
  7    1.7771  # MTAU
BLOCK MINPAR
  3     5.     # TANBETA
BLOCK EXTPAR
  1    5.D2    # M1
  2    1.D3    # M2
  3    3.D3    # M3
  11   1.5D3   # ATOP
  12   1.5D3   # ABOT
  13   1.5D3   # ATAU
  23   180.    # MU
  31   1.D3    # LEFT SELECTRON
  32   1.D3    # LEFT SMUON
  33   1.D3    # LEFT STAU
  34   1.D3    # RIGHT SELECTRON
  35   1.D3    # RIGHT SMUON
  36   1.D3    # RIGHT STAU
  41   1.D3    # LEFT 1ST GEN. SQUARKS
  42   1.D3    # LEFT 2ND GEN. SQUARKS
  43   1.D3    # LEFT 3RD GEN. SQUARKS
  44   1.D3    # RIGHT U-SQUARKS
  45   1.D3    # RIGHT C-SQUARKS
  46   1.D3    # RIGHT T-SQUARKS
  47   1.D3    # RIGHT D-SQUARKS
  48   1.D3    # RIGHT S-SQUARKS
  49   1.D3    # RIGHT B-SQUARKS
  61   .3D0    # LAMBDA
  62   .3D0    # KAPPA
  63   200.    # A_LAMBDA
  64   0.0     # A_KAPPA
```

Table 1: Sample `slhainp.dat` file.

```
# NMHDECAY OUTPUT IN SLHA FORMAT
# Info about spectrum calculator
BLOCK SPINFO        # Program information
     1   NMHDECAY   # spectrum calculator
     2   1.1        # version number
# Input parameters
BLOCK MODSEL
    3     1         # NMSSM PARTICLE CONTENT
BLOCK SMINPUTS
     1     1.37036000E+02   # ALPHA_EM^-1
     2     1.16639000E-05   # GF
     3     1.20000000E-01   # ALPHA_S(MZ)
     4     9.11870000E+01   # MZ
     5     4.24000000E+00   # MB(MB)
     6     1.75000000E+02   # MTOP (POLE MASS)
     7     1.77710000E+00   # MTAU
# SMINPUTS Beyond SLHA:
# MW:     0.80419998E+02
# MS:     0.19000000E+00
# MC:     0.16100000E+01
# VUS:     0.22200000E+00
# VCB:     0.41000000E-01
# VUB:     0.36000000E-02
BLOCK MINPAR
     3     5.00000000E+00   # TANBETA
BLOCK EXTPAR
     1     5.00000000E+02   # M1
     2     1.00000000E+03   # M2
     3     3.00000000E+03   # M3
    11     1.50000000E+03   # ATOP
    12     1.50000000E+03   # ABOTTOM
    13     1.50000000E+03   # ATAU
    23     1.80000000E+02   # MU
    33     1.00000000E+03   # LEFT STAU
    36     1.00000000E+03   # RIGHT STAU
    43     1.00000000E+03   # LEFT 3RD GEN. SQUARKS
    46     1.00000000E+03   # RIGHT T-SQUARKS
    49     1.00000000E+03   # RIGHT B-SQUARKS
    61     3.00000000E-01   # LAMBDA
    62     3.00000000E-01   # KAPPA
    63     2.00000000E+02   # A_LAMBDA
    64     0.00000000E+00   # A_KAPPA
#
BLOCK MASS   # Mass spectrum
#  PDG Code     mass              particle
        25     1.16801961E+02   # lightest neutral scalar
        35     3.56992533E+02   # second neutral scalar
        45     5.91271441E+02   # third neutral scalar
        36     4.90592823E+01   # lightest pseudoscalar
        46     5.87736637E+02   # second pseudoscalar
        37     5.90714771E+02   # charged Higgs
   1000022     1.66456703E+02   # neutralino(1)
   1000023    -1.85971845E+02   # neutralino(2)
   1000025     3.67762262E+02   # neutralino(3)
   1000035     5.04634098E+02   # neutralino(4)
   1000045     1.00711878E+03   # neutralino(5)
   1000024     1.76259627E+02   # chargino(1)
   1000037     1.00710838E+03   # chargino(2)
   1000005     8.21132887E+02   #  ~b_1
   1000006     6.24445725E+02   #  ~t_1
   2000005     8.24131926E+02   #  ~b_2
   2000006     1.00488215E+03   #  ~t_2
```

```
# 3*3 Higgs mixing
BLOCK HIGMIX
  1  1    -1.27437514E-01   # S_(1,1)
  1  2    -3.16386555E-02   # S_(1,2)
  1  3    -9.91341856E-01   # S_(1,3)
  2  1    -2.01793548E-01   # S_(2,1)
  2  2    -9.77759539E-01   # S_(2,2)
  2  3     5.71458423E-02   # S_(2,3)
  3  1    -9.71101974E-01   # S_(3,1)
  3  2     2.07328914E-01   # S_(3,2)
  3  3     1.18218769E-01   # S_(3,3)
# 2*2 Pseudoscalar Higgs mixing
BLOCK AMIX
  1  1    -2.37325986E-02   # P'_(1,1)
  1  2    -9.99718342E-01   # P'_(1,2)
  2  1     9.99718342E-01   # P'_(2,1)
  2  2    -2.37325986E-02   # P'_(2,2)
# Gaugino-Higgsino mixing
BLOCK NMIX  # 5*5 Neutralino Mixing Matrix
  1  1     1.05202671E-01   # N_(1,1)
  1  2    -7.87589631E-02   # N_(1,2)
  1  3     6.92565025E-01   # N_(1,3)
  1  4    -6.93980327E-01   # N_(1,4)
  1  5     1.46541512E-01   # N_(1,5)
  2  1    -3.48143940E-02   # N_(2,1)
  2  2     3.76738427E-02   # N_(2,2)
  2  3     7.01102969E-01   # N_(2,3)
  2  4     7.06807991E-01   # N_(2,4)
  2  5     7.90299289E-02   # N_(2,5)
  3  1    -2.45218320E-02   # N_(3,1)
  3  2     9.59570918E-03   # N_(3,2)
  3  3    -1.58479313E-01   # N_(3,3)
  3  4     4.52366830E-02   # N_(3,4)
  3  5     9.85973910E-01   # N_(3,5)
  4  1     9.93510538E-01   # N_(4,1)
  4  2     1.73884449E-02   # N_(4,2)
  4  3    -5.24484443E-02   # N_(4,3)
  4  4     9.87396900E-02   # N_(4,4)
  4  5     1.15796113E-02   # N_(4,5)
  5  1    -7.47306864E-03   # N_(5,1)
  5  2     9.95983570E-01   # N_(5,2)
  5  3     3.06884365E-02   # N_(5,3)
  5  4    -8.37728195E-02   # N_(5,4)
  5  5    -1.10280322E-03   # N_(5,5)
BLOCK UMIX  # Chargino U Mixing Matrix
  1  1     4.31428408E-02   # U_(1,1)
  1  2    -9.99068914E-01   # U_(1,2)
  2  1     9.99068914E-01   # U_(2,1)
  2  2     4.31428408E-02   # U_(2,2)
BLOCK VMIX  # Chargino V Mixing Matrix
  1  1     1.18343118E-01   # V_(1,1)
  1  2    -9.92972762E-01   # V_(1,2)
  2  1     9.92972762E-01   # V_(2,1)
  2  2     1.18343118E-01   # V_(2,2)
```

Table 2: Corresponding spectr.dat output file.

```
#
#  Total number of points scanned
#
1000
#
#  Output format 0=short 1=long (not recommended for big scannings)
#
0
#
#  lambda
#
0.5
0.5
#
#  kappa
#
-0.15
-0.15
#
#  tan(beta)
#
3.5
3.5
#
#  mu
#
200.
200.
#
#  A_lambda
#
780.
780.
#
#  A_kappa
#
150.0
250.0
#
#  Remaining soft terms (no scan)
#
mQ3=  1.D3
mU3=  1.D3
mD3=  1.D3
mL3=  1.D3
mE3=  1.D3
AU3=  1.5D3
AD3=  1.5D3
AE3=  1.5D3
mQ=   1.D3
mU=   1.D3
mD=   1.D3
mL=   1.D3
mE=   1.D3
M1=   5.D2
M2=   1.D3
M3=   3.D3
```

Table 3: The `scaninp.dat` file for parameter scan #2.

A. Conventions and tree level mass matrices

A.1 General conventions for the NMSSM

Below, we define our conventions for the tree level lagrangian of the NMSSM. The superpotential for the Higgs fields, the quarks and the leptons of the 3rd generation is

$$W = h_t \widehat{Q} \cdot \widehat{H}_u \widehat{T}_R^c - h_b \widehat{Q} \cdot \widehat{H}_d \widehat{B}_R^c - h_\tau \widehat{L} \cdot \widehat{H}_d \widehat{L}_R^c + \lambda \widehat{S} \widehat{H}_u \cdot \widehat{H}_d + \frac{1}{3} \kappa \widehat{S}^3 \,. \tag{A.1}$$

Hereafter, hatted capital letters denote superfields, and unhatted capital letters the corresponding (complex) scalar components. The SU(2) doublets are

$$\widehat{Q} = \begin{pmatrix} \widehat{T}_L \\ \widehat{B}_L \end{pmatrix}, \quad \widehat{L} = \begin{pmatrix} \widehat{\nu}_{\tau L} \\ \widehat{\tau}_L \end{pmatrix}, \quad \widehat{H}_u = \begin{pmatrix} \widehat{H}_u^+ \\ \widehat{H}_u^0 \end{pmatrix}, \quad \widehat{H}_d = \begin{pmatrix} \widehat{H}_d^0 \\ \widehat{H}_d^- \end{pmatrix}. \tag{A.2}$$

Products of two SU(2) doublets are defined as, e.g.,

$$\widehat{H}_u \cdot \widehat{H}_d = \widehat{H}_u^+ \widehat{H}_d^- - \widehat{H}_u^0 \widehat{H}_d^0 \,. \tag{A.3}$$

For the soft Susy breaking terms we take

$$\begin{aligned} -\mathcal{L}_{\mathrm{soft}} = {} & m_{\mathrm{H_u}}^2 |H_u|^2 + m_{\mathrm{H_d}}^2 |H_d|^2 + m_{\mathrm{S}}^2 |S|^2 + m_Q^2 |Q^2| + m_T^2 |T_R^2| + \\ & + m_B^2 |B_R^2| + m_L^2 |L^2| + m_\tau^2 |L_R^2| + \\ & + \Big(h_t A_t \; Q \cdot H_u T_R^c - h_b A_b \; Q \cdot H_d B_R^c - h_\tau A_\tau \; L \cdot H_d L_R^c \\ & \quad + \lambda A_\lambda \; H_u \cdot H_d S + \frac{1}{3} \kappa A_\kappa \; S^3 + \mathrm{h.c.} \Big) \,. \end{aligned} \tag{A.4}$$

A.2 Higgs sector at tree level

For completeness, we list here the Higgs potential, tree level Higgs masses and our conventions for the mixing angles. The tree level Higgs potential is given by

$$\begin{aligned} V = {} & \lambda^2 (|H_u|^2 |S|^2 + |H_d|^2 |S|^2 + |H_u \cdot H_d|^2) + \kappa^2 |S^2|^2 + \\ & + \lambda\kappa (H_u \cdot H_d S^{*2} + \mathrm{h.c.}) + \frac{1}{4} g^2 (|H_u|^2 - |H_d|^2)^2 + \\ & + \frac{1}{2} g_2^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 + m_{\mathrm{H_u}}^2 |H_u|^2 + m_{\mathrm{H_d}}^2 |H_d|^2 + m_S^2 |S|^2 + \\ & + (\lambda A_\lambda H_u \cdot H_d S + \frac{1}{3} \kappa A_\kappa \; S^3 + \mathrm{h.c.}) \,, \end{aligned} \tag{A.5}$$

where

$$g^2 = \frac{1}{2} (g_1^2 + g_2^2) \,. \tag{A.6}$$

Assuming vevs h_u, h_d and s such that

$$H_u^0 = h_u + \frac{H_{uR} + iH_{uI}}{\sqrt{2}} \,, \quad H_d^0 = h_d + \frac{H_{dR} + iH_{dI}}{\sqrt{2}} \,, \quad S = s + \frac{S_R + iS_I}{\sqrt{2}} \tag{A.7}$$

eq. (A.5) simplifies to

$$
\begin{aligned}
V =\ & \lambda^2(h_u^2 s^2 + h_d^2 s^2 + h_u^2 h_d^2) + \kappa^2 s^4 - 2\lambda\kappa h_u h_d s^2 - 2\lambda A_\lambda\, h_u h_d s + \\
& + \frac{2}{3}\,\kappa A_\kappa s^3 + m_{\mathrm{H_u}}^2 h_u^2 + m_{\mathrm{H_d}}^2 h_d^2 + m_S^2 s^2 + \frac{1}{4}\, g^2 (h_u^2 - h_d^2)^2\,. \qquad \text{(A.8)}
\end{aligned}
$$

The sign conventions for the fields can be chosen such that the Yukawa couplings λ, h_t, h_b, the vevs h_u, h_d (and hence $\tan\beta$) as well as the soft gaugino masses M_i are all positive. Then, the Yukawa coupling κ, the trilinear soft terms A_i, and the vev s (and hence μ_{eff}) can all be either positive or negative.

A.3 CP-even neutral states

In the basis $S^{bare} = (H_{uR}, H_{dR}, S_R)$ and using the minimization equations in order to eliminate the soft masses squared, one obtains the following mass-squared matrix entries:

$$
\begin{aligned}
\mathcal{M}^2_{S,11} &= g^2 h_u^2 + \lambda s \frac{h_d}{h_u}\,(A_\lambda + \kappa s)\,, \\
\mathcal{M}^2_{S,22} &= g^2 h_d^2 + \lambda s \frac{h_u}{h_d}\,(A_\lambda + \kappa s)\,, \\
\mathcal{M}^2_{S,33} &= \lambda A_\lambda \frac{h_u h_d}{s} + \kappa s (A_\kappa + 4\kappa s)\,, \\
\mathcal{M}^2_{S,12} &= (2\lambda^2 - g^2) h_u h_d - \lambda s (A_\lambda + \kappa s)\,, \\
\mathcal{M}^2_{S,13} &= 2\lambda^2 h_u s - \lambda h_d (A_\lambda + 2\kappa s)\,, \\
\mathcal{M}^2_{S,23} &= 2\lambda^2 h_d s - \lambda h_u (A_\lambda + 2\kappa s)\,. \qquad \text{(A.9)}
\end{aligned}
$$

After diagonalization by an orthogonal matrix S_{ij} one obtains 3 CP-even states (ordered in mass) $h_i = S_{ij} S_j^{bare}$, with masses denoted by m_{h_i}.

In the MSSM limit (λ, $\kappa \to 0$, and parameters such that $h_3 \sim S_R$) the elements of the first 2×2 sub-matrix of S_{ij} are related to the MSSM angle α as

$$
\begin{aligned}
S_{11} &\sim \ \cos\alpha\,, & S_{21} &\sim \sin\alpha\,, \\
S_{12} &\sim -\sin\alpha\,, & S_{22} &\sim \cos\alpha\,. \qquad \text{(A.10)}
\end{aligned}
$$

A.4 CP-odd neutral states

In the basis $P^{\mathrm{bare}} = (H_{uI}, H_{dI}, S_I)$ and using the minimization equations in order to eliminate the soft masses squared, one obtains the following mass-squared matrix entries:

$$
\begin{aligned}
\mathcal{M}^2_{P,11} &= \lambda s \frac{h_d}{h_u}\,(A_\lambda + \kappa s)\,, \\
\mathcal{M}^2_{P,22} &= \lambda s \frac{h_u}{h_d}\,(A_\lambda + \kappa s)\,, \\
\mathcal{M}^2_{P,33} &= 4\lambda\kappa h_u h_d + \lambda A_\lambda \frac{h_u h_d}{s} - 3\kappa A_\kappa s\,, \\
\mathcal{M}^2_{P,12} &= \lambda s (A_\lambda + \kappa s)\,, \\
\mathcal{M}^2_{P,13} &= \lambda h_d (A_\lambda - 2\kappa s)\,, \\
\mathcal{M}^2_{P,23} &= \lambda h_u (A_\lambda - 2\kappa s)\,. \qquad \text{(A.11)}
\end{aligned}
$$

The diagonalization of this mass matrix is performed in two steps. First, one rotates into a basis $(\tilde{A}, \tilde{G}, S_I)$, where $\tilde{G}$ is a massless Goldstone mode:

$$\begin{pmatrix} H_{uI} \\ H_{dI} \\ S_I \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{A} \\ \tilde{G} \\ S_I \end{pmatrix}, \tag{A.12}$$

where $\tan\beta = h_u/h_d$. Dropping the Goldstone mode, the remaining 2×2 mass matrix in the basis $(\tilde{A}, S_I)$ has the matrix elements

$$\begin{aligned} \mathcal{M}^2_{P,11} &= \lambda s \frac{h_u^2 + h_d^2}{h_u h_d} (A_\lambda + \kappa s)\,, \\ \mathcal{M}^2_{P,22} &= 4\lambda\kappa h_u h_d + \lambda A_\lambda \frac{h_u h_d}{s} - 3\kappa A_\kappa s\,, \\ \mathcal{M}^2_{P,12} &= \lambda\sqrt{h_u^2 + h_d^2}\,(A_\lambda - 2\kappa s)\,. \end{aligned} \tag{A.13}$$

It can be diagonalized by an orthogonal 2×2 matrix P'_{ij} such that the physical CP-odd states a_i (ordered in mass) are

$$\begin{aligned} a_1 &= P'_{11}\tilde{A} + P'_{12}S_I \\ &= P'_{11}(\cos\beta H_{uI} + \sin\beta H_{dI}) + P'_{12}S_I\,, \\ a_2 &= P'_{21}\tilde{A} + P'_{22}S_I \\ &= P'_{21}(\cos\beta H_{uI} + \sin\beta H_{dI}) + P'_{22}S_I\,, \end{aligned} \tag{A.14}$$

and, for completeness,

$$\tilde{G} = -\sin\beta H_{uI} + \cos\beta H_{dI}\,. \tag{A.15}$$

The decomposition of the bare states in terms of physical states reads

$$\begin{aligned} H_{uI} &= \cos\beta(P'_{11}a_1 + P'_{21}a_2) - \sin\beta\tilde{G}\,, \\ H_{dI} &= \sin\beta(P'_{11}a_1 + P'_{21}a_2) + \cos\beta\tilde{G}\,, \\ S_I &= P'_{12}a_1 + P'_{22}a_2\,. \end{aligned} \tag{A.16}$$

(In principle, since the matrix P'_{ij} is orthogonal, it could be parameterized by one angle.) eqs. (A.16) suggest the introduction of a 2×3 matrix P_{ij} with

$$P_{i1} = \cos\beta P'_{i1}, P_{i2} = \sin\beta P'_{i1}, P_{i3} = P'_{i2} \tag{A.17}$$

such that, omitting the Goldstone boson,

$$\begin{aligned} H_{uI} &= P_{11}a_1 + P_{21}a_2\,, \\ H_{dI} &= P_{11}a_1 + P_{21}a_2\,, \\ S_I &= P_{13}a_1 + P_{23}a_2\,. \end{aligned} \tag{A.18}$$

A.5 Charged states

In the basis $(H_u^+, [H_d^-]^* = H_d^+)$, the charged Higgs mass matrix is given by

$$\mathcal{M}_\pm^2 = \left(\lambda s(A_\lambda + \kappa s) + h_u h_d \left(\frac{g_2^2}{2} - \lambda^2\right)\right) \begin{pmatrix} \cot\beta & 1 \\ 1 & \tan\beta \end{pmatrix}. \tag{A.19}$$

This gives one eigenstate $H^\pm$ of mass $\mathrm{Tr}\mathcal{M}_\pm^2$ and one massless goldstone mode $G^\pm$ with

$$\begin{aligned} H_u^\pm &= \cos\beta H^\pm - \sin\beta G^\pm\,, \\ H_d^\pm &= \sin\beta H^\pm + \cos\beta G^\pm\,. \end{aligned} \tag{A.20}$$

A.6 SUSY particles

A.6.1 Neutralinos

Denoting the $U(1)_Y$ gaugino by λ_1 and the neutral SU(2) gaugino by λ_2^3, the mass terms in the lagrangian read

$$\begin{aligned} \mathcal{L} = &\frac{1}{2}\,M_1\lambda_1\lambda_1 + \frac{1}{2}\,M_2\lambda_2^3\lambda_2^3 + \\ &+ \lambda(s\psi_u^0\psi_d^0 + h_u\psi_d^0\psi_s + h_d\psi_u^0\psi_s) - \kappa s\psi_s\psi_s + \\ &+ \frac{ig_1}{\sqrt{2}}\,\lambda_1(h_u\psi_u^0 - h_d\psi_d^0) - \frac{ig_2}{\sqrt{2}}\,\lambda_2^3(h_u\psi_u^0 - h_d\psi_d^0)\,. \end{aligned} \tag{A.21}$$

In the basis $\psi^0 = (-i\lambda_1, -i\lambda_2, \psi_u^0, \psi_d^0, \psi_s)$ one can rewrite

$$\mathcal{L} = -\frac{1}{2}\,(\psi^0)^T\mathcal{M}_0(\psi^0) + \text{h.c.}\,, \tag{A.22}$$

where

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & \frac{g_1 h_u}{\sqrt{2}} & -\frac{g_1 h_d}{\sqrt{2}} & 0 \\ & M_2 & -\frac{g_2 h_u}{\sqrt{2}} & \frac{g_2 h_d}{\sqrt{2}} & 0 \\ & & 0 & -\mu & -\lambda h_d \\ & & & 0 & -\lambda h_u \\ & & & & 2\kappa s \end{pmatrix}. \tag{A.23}$$

(Recall that here $\mu = \mu_{\text{eff}} = \lambda s$). One obtains 5 eigenstates (ordered in mass) $\chi_i^0 = N_{ij}\psi_j^0$, with N_{ij} real, with masses $m_{\chi_i^0}$ that are real, but not necessarily positive.

A.6.2 Charginos

The charged SU(2) gauginos are $\lambda^- = \frac{1}{\sqrt{2}}(\lambda_2^1 + i\lambda_2^2)$, $\lambda^+ = \frac{1}{\sqrt{2}}(\lambda_2^1 - i\lambda_2^2)$. Defining

$$\psi^+ = \begin{pmatrix} -i\lambda^+ \\ \psi_u^+ \end{pmatrix}, \qquad \psi^- = \begin{pmatrix} -i\lambda^- \\ \psi_d^- \end{pmatrix} \tag{A.24}$$

the lagrangian can be written as

$$\mathcal{L} = -\frac{1}{2}\,(\psi^+, \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c.} \tag{A.25}$$

with

$$X = \begin{pmatrix} M_2 & g_2 h_u \\ g_2 h_d & \mu \end{pmatrix}. \tag{A.26}$$

The mass eigenstates are $\chi^+ = V\psi^+$, $\chi^- = U\psi^-$, with

$$U = \begin{pmatrix} \cos\theta_U & \sin\theta_U \\ -\sin\theta_U & \cos\theta_U \end{pmatrix}, \qquad V = \begin{pmatrix} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{pmatrix}. \tag{A.27}$$

Defining

$$\gamma = \sqrt{\mathrm{Tr}(X^T X)^2 - 4\mathrm{det}(X^T X)} \tag{A.28}$$

one has

$$\begin{aligned} \tan\theta_U &= \frac{g_2^2(h_d^2 - h_u^2) + \mu^2 - M_2^2 - \gamma}{2g_2(M_2 h_u + \mu h_d)}, \\ \tan\theta_V &= \frac{g_2^2(h_u^2 - h_d^2) + \mu^2 - M_2^2 - \gamma}{2g_2(M_2 h_d + \mu h_u)}, \end{aligned} \tag{A.29}$$

where $-\pi/2 \leq \theta_U, \theta_V \leq \pi/2$ are such that $M_D = UXV^T$ is diagonal, but not necessarily positive. The masses with $|m_{\widetilde{\chi}_1}| < |m_{\widetilde{\chi}_2}|$ are given by

$$\begin{aligned} m_{\widetilde{\chi}_1} &= \cos\theta_U(M_2\cos\theta_V + g_2 h_u \sin\theta_V) + \sin\theta_U(g_2 h_d \cos\theta_V + \mu\sin\theta_V), \\ m_{\widetilde{\chi}_2} &= \sin\theta_U(M_2\sin\theta_V - g_2 h_u \cos\theta_V) - \cos\theta_U(g_2 h_d \sin\theta_V - \mu\cos\theta_V). \end{aligned} \tag{A.30}$$

In terms of 4 component Dirac spinors $\Psi_i = \begin{pmatrix} \chi_i^+ \\ \overline{\chi}_i^- \end{pmatrix}$ one can rewrite the lagrangian as

$$\mathcal{L} = -\chi^- M_D \chi^+ + \text{h.c.} = -m_{\widetilde{\chi}_1}\overline{\Psi}_1\Psi_1 - m_{\widetilde{\chi}_2}\overline{\Psi}_2\Psi_2. \tag{A.31}$$

A.7 Top and bottom squarks

To complete the consequences of our conventions above, we give here the top and bottom squark mass-squared matrices (without the D-term contributions). Below, t_L, t_R^c, b_L and b_R^c denote the two component quark spinors.

Top squarks:

$$\begin{array}{c} \\ T_R^* \\ T_L^* \end{array} \begin{array}{c} \begin{array}{cc} \quad T_R \quad\quad & \quad\quad T_L \end{array} \\ \begin{pmatrix} m_T^2 + h_t^2 h_u^2 & h_t(A_t h_u - \lambda s h_d) \\ h_t(A_t h_u - \lambda s h_d) & m_Q^2 + h_t^2 h_u^2 \end{pmatrix} \end{array}. \tag{A.32}$$

Bottom squarks:

$$\begin{array}{c} \\ B_R^* \\ B_L^* \end{array} \begin{array}{c} \begin{array}{cc} \quad B_R \quad\quad & \quad\quad B_L \end{array} \\ \begin{pmatrix} m_B^2 + h_b^2 h_d^2 & h_b(A_b h_d - \lambda s h_u) \\ h_b(A_b h_d - \lambda s h_u) & m_Q^2 + h_b^2 h_d^2 \end{pmatrix} \end{array}. \tag{A.33}$$

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B. Tree level Feynman rules

B.1 Feynman rules for the Higgs couplings

B.1.1 Higgs-quarks

The couplings are obtained by expanding the quark mass matrices in the (properly normalized) physical Higgs fields h_i, a_i and $H^\pm$. Below, we use $v^2 = h_u^2 + h_d^2$, and consider the quarks of the third generation.

$$\begin{aligned}
h_i t_L t_R^c &: \frac{m_t}{\sqrt{2} v \sin\beta} S_{i1} \\
h_i b_L b_R^c &: \frac{m_b}{\sqrt{2} v \cos\beta} S_{i2} \\
a_i t_L t_R^c &: i\frac{m_t}{\sqrt{2} v \sin\beta} P_{i1} \\
a_i b_L b_R^c &: i\frac{m_b}{\sqrt{2} v \cos\beta} P_{i2} \\
H^+ b_L t_R^c &: -\frac{m_t}{v}\cot\beta \\
H^- t_L b_R^c &: -\frac{m_b}{v}\tan\beta
\end{aligned} \tag{B.1}$$

B.2 Higgs-Gauge bosons

These couplings are obtained from the kinetic terms in the lagrangian:

$$\begin{aligned}
h_i Z_\mu Z_\nu &: g_{\mu\nu}\frac{g_1^2+g_2^2}{\sqrt{2}}(h_u S_{i1} + h_d S_{i2}) \\
h_i W_\mu^+ W_\nu^- &: g_{\mu\nu}\frac{g_2^2}{\sqrt{2}}(h_u S_{i1} + h_d S_{i2}) \\
h_i(p) H^+(p') W_\mu^- &: \frac{g_2}{2}(\cos\beta S_{i1} - \sin\beta S_{i2})(p-p')_\mu \\
a_i(p) H^+(p') W_\mu^- &: i\frac{g_2}{2}(\cos\beta P_{i1} + \sin\beta P_{i2})(p-p')_\mu \\
h_i(p) a_j(p') Z_\mu &: i\frac{g}{\sqrt{2}}(S_{i1}P_{j1} - S_{i2}P_{j2})(p-p')_\mu \\
H^+(p) H^-(p') Z_\mu &: \frac{g_1^2 - g_2^2}{\sqrt{g_1^2+g_2^2}}(p-p')_\mu
\end{aligned} \tag{B.2}$$

B.3 Higgs-neutralinos/charginos

As in the case of the Higgs-Quark couplings, these couplings are obtained by expanding the corresponding mass matrices:

$$\begin{aligned}
h_a \chi_i^+ \chi_j^- &: \frac{\lambda}{\sqrt{2}} S_{a3} U_{i2} V_{j2} + \frac{g_2}{\sqrt{2}}(S_{a1} U_{i1} V_{j2} + S_{a2} U_{i2} V_{j1}) \\
a_a \chi_i^+ \chi_j^- &: i\left(\frac{\lambda}{\sqrt{2}} P_{a3} U_{i2} V_{j2} - \frac{g_2}{\sqrt{2}}(P_{a1} U_{i1} V_{j2} + P_{a2} U_{i2} V_{j1})\right) \\
H^+ \chi_i^- \chi_j^0 &: \lambda\cos\beta U_{i2} N_{j5} - \frac{\sin\beta}{\sqrt{2}} U_{i2}(g_1 N_{j1} + g_2 N_{j2}) + g_2 \sin\beta U_{i1} N_{j4}
\end{aligned}$$

$$\begin{aligned}
H^-\chi_i^+\chi_j^0 &: \lambda\sin\beta V_{i2}N_{j5} + \frac{\cos\beta}{\sqrt{2}}V_{i2}(g_1N_{j1}+g_2N_{j2}) + g_2\cos\beta V_{i1}N_{j3} \\
h_a\chi_i^0\chi_j^0 &: \frac{\lambda}{\sqrt{2}}(S_{a1}\Pi_{ij}^{45} + S_{a2}\Pi_{ij}^{35} + S_{a3}\Pi_{ij}^{34}) - \sqrt{2}\kappa S_{a3}N_{i5}N_{j5} - \\
&\quad - \frac{g_1}{2}(S_{a1}\Pi_{ij}^{13} - S_{a2}\Pi_{ij}^{14}) + \frac{g_2}{2}(S_{a1}\Pi_{ij}^{23} - S_{a2}\Pi_{ij}^{24}) \\
a_a\chi_i^0\chi_j^0 &: i\bigg(\frac{\lambda}{\sqrt{2}}(P_{a1}\Pi_{ij}^{45} + P_{a2}\Pi_{ij}^{35} + P_{a3}\Pi_{ij}^{34}) - \sqrt{2}\kappa P_{a3}N_{i5}N_{j5} + \\
&\quad + \frac{g_1}{2}(P_{a1}\Pi_{ij}^{13} - P_{a2}\Pi_{ij}^{14}) - \frac{g_2}{2}(P_{a1}\Pi_{ij}^{23} - P_{a2}\Pi_{ij}^{24})\bigg)\,, \qquad \text{(B.3)}
\end{aligned}$$

where $\Pi_{ij}^{ab} = N_{ia}N_{jb} + N_{ib}N_{ja}$.

B.4 Triple Higgs interactions

The trilinear Higgs self-couplings are obtained by expanding the scalar potential.

$$\begin{aligned}
h_ah_bh_c &: \frac{\lambda^2}{\sqrt{2}}\left(h_u(\Pi_{abc}^{122} + \Pi_{abc}^{133}) + h_d(\Pi_{abc}^{211} + \Pi_{abc}^{233}) + s(\Pi_{abc}^{311} + \Pi_{abc}^{322})\right) - \\
&\quad - \frac{\lambda\kappa}{\sqrt{2}}(h_u\Pi_{abc}^{323} + h_d\Pi_{abc}^{313} + 2s\Pi_{abc}^{123}) + \sqrt{2}\kappa^2 s\Pi_{abc}^{333} - \\
&\quad - \frac{\lambda A_\lambda}{\sqrt{2}}\Pi_{abc}^{123} + \frac{\kappa A_\kappa}{3\sqrt{2}}\Pi_{abc}^{333} + \\
&\quad + \frac{g^2}{2\sqrt{2}}\left(h_u(\Pi_{abc}^{111} - \Pi_{abc}^{122}) - h_d(\Pi_{abc}^{211} - \Pi_{abc}^{222})\right)\,, \qquad \text{(B.4)}
\end{aligned}$$

where

$$\begin{aligned}
\Pi_{abc}^{ijk} &= S_{ai}S_{bj}S_{ck} + S_{ai}S_{cj}S_{bk} + S_{bi}S_{aj}S_{ck} + \\
&\quad + S_{bi}S_{cj}S_{ak} + S_{ci}S_{aj}S_{bk} + S_{ci}S_{bj}S_{ak}\,. \qquad \text{(B.5)} \\
h_aa_ba_c &: \frac{\lambda^2}{\sqrt{2}}\left(h_u(\Pi_{abc}^{122} + \Pi_{abc}^{133}) + h_d(\Pi_{abc}^{211} + \Pi_{abc}^{233}) + s(\Pi_{abc}^{311} + \Pi_{abc}^{322})\right) + \\
&\quad + \frac{\lambda\kappa}{\sqrt{2}}\bigg(h_u(\Pi_{abc}^{233} - 2\Pi_{abc}^{323}) + h_d(\Pi_{abc}^{133} - 2\Pi_{abc}^{313}) + \\
&\qquad\qquad + 2s(\Pi_{abc}^{312} - \Pi_{abc}^{123} - \Pi_{abc}^{213})\bigg) + \sqrt{2}\kappa^2 s\Pi_{abc}^{333} + \\
&\quad + \frac{\lambda A_\lambda}{\sqrt{2}}(\Pi_{abc}^{123} + \Pi_{abc}^{213} + \Pi_{abc}^{312}) - \frac{\kappa A_\kappa}{\sqrt{2}}\Pi_{abc}^{333} + \\
&\quad + \frac{g^2}{2\sqrt{2}}\left(h_u(\Pi_{abc}^{111} - \Pi_{abc}^{122}) - h_d(\Pi_{abc}^{211} - \Pi_{abc}^{222})\right)\,, \qquad \text{(B.6)}
\end{aligned}$$

where

$$\begin{aligned}
\Pi_{abc}^{ijk} &= S_{ai}(P_{bj}P_{ck} + P_{cj}P_{bk})\,. \qquad \text{(B.7)} \\
h_aH^+H^- &: \frac{\lambda^2}{\sqrt{2}}(s(\Pi_a^{311} + \Pi_a^{322}) - h_u\Pi_a^{212} - h_d\Pi_a^{112}) + \\
&\quad + \sqrt{2}\lambda\kappa s\Pi_a^{312} + \frac{\lambda A_\lambda}{\sqrt{2}}\Pi_a^{312} +
\end{aligned}$$

$$+\frac{g_1^2}{4\sqrt{2}}\left(h_u(\Pi_a^{111}-\Pi_a^{122})+h_d(\Pi_a^{222}-\Pi_a^{211})\right)+$$
$$+\frac{g_2^2}{4\sqrt{2}}\left(h_u(\Pi_a^{111}+\Pi_a^{122}+2\Pi_a^{212})+h_d(\Pi_a^{211}+\Pi_a^{222}+2\Pi_a^{112})\right), \quad \text{(B.8)}$$

where

$$\Pi_a^{ijk}=2S_{ai}C_jC_k \quad \text{(B.9)}$$

with $C_1=\cos\beta$, $C_2=\sin\beta$.

C. Radiative corrections to the Higgs mass matrices

In this appendix we give the explicit expressions of the radiative corrections to the Higgs masses.

Generally, the radiative corrections to the Higgs effective action consist in corrections to the wave function renormalization constants Z_i, and corrections to the effective potential $V_{\mathrm{eff}}(h_i)$. We will treat the one loop corrections originating from stop and sbottom loops exactly in the stop and sbottom mixings and mass splittings. The resulting contributions to the Higgs masses are nevertheless quite simple if they are expressed in terms of couplings and masses at the scale $Q=M_{SUSY}$ (the average of the squark masses, see below) and in terms of the Higgs vevs *before* rescaling by $Z_i^{1/2}$, which we will denote by $h_i(Q)$. However, the Higgs vevs are related to the Fermi coupling G_F and our definition of $\tan\beta$ *after* rescaling by $Z_i^{1/2}$, cf. eq. (2.9).

Hence, we first have to determine Z_i for $i=H_u,H_d$:

$$\begin{aligned} Z_{H_u} &= 1+\frac{3h_t^2}{16\pi^2}t \\ Z_{H_d} &= 1+\frac{3h_b^2}{16\pi^2}t\,, \end{aligned} \quad \text{(C.1)}$$

where

$$\begin{aligned} t &= \ln\left(\frac{Q^2}{m_{\mathrm{top}}^2}\right), \\ Q^2 &= \frac{1}{4}(2m_Q^2+m_T^2+m_B^2)+m_{\mathrm{top}}^2 \end{aligned} \quad \text{(C.2)}$$

(such that $t=0$ for vanishing SUSY breaking squark masses m_Q, m_T and m_B). Now we have

$$h_u(Q)=\frac{h_u}{\sqrt{Z_{H_u}}}, \qquad h_d(Q)=\frac{h_d}{\sqrt{Z_{H_u}}}. \quad \text{(C.3)}$$

with h_u and h_d given in terms of G_F and $\tan\beta$.

In addition we need the running Yukawa couplings and quark masses at the scale Q. First, the running bottom quark mass (and hence h_b) at the scale m_{top} is given by its QCD evolution to one loop order

$$h_b(m_{\mathrm{top}})=h_b(m_{\mathrm{bot}})\left(1-\frac{23}{12\pi}\alpha_s(m_{\mathrm{top}})\ln\frac{m_{\mathrm{top}}^2}{m_{\mathrm{bot}}^2}\right)^{12/23}. \quad \text{(C.4)}$$

The running top quark Yukawa coupling at the scale $m_{\rm top}$ is given in terms of the top quark pole mass as[2]

$$h_t(m_{\rm top}) = \frac{m_{\rm top}(pole)}{h_u}\left(1 + \frac{4\alpha_s(m_{\rm top})}{3\pi} + \frac{11\alpha_s^2(m_{\rm top})}{\pi^2}\right). \tag{C.5}$$

Next, the running Yukawa couplings at the scale Q are obtained from the RG equations (for an effective 2 Higgs doublet model)

$$\begin{aligned} \frac{dh_t}{d\ln Q^2} &= \frac{h_t}{64\pi^2}(9h_t^2 + h_b^2) - h_t\frac{\alpha_s}{\pi}\,, \\ \frac{dh_b}{d\ln Q^2} &= \frac{h_b}{64\pi^2}(9h_b^2 + h_t^2) - h_b\frac{\alpha_s}{\pi}\,. \end{aligned} \tag{C.6}$$

Its solutions, exact in α_s but perturbative in the Yukawa couplings, are

$$\begin{aligned} h_t(Q) &= h_t(m_{\rm top})\left(1 + \frac{7}{4\pi}\alpha_s(m_{\rm top})t\right)^{-4/7}\left(1 + \frac{1}{64\pi^2}(9h_t^2 + h_b^2)t\right)\,, \\ h_b(Q) &= h_b(m_{\rm top})\left(1 + \frac{7}{4\pi}\alpha_s(m_{\rm top})t\right)^{-4/7}\left(1 + \frac{1}{64\pi^2}(9h_b^2 + h_t^2)t\right)\,. \end{aligned} \tag{C.7}$$

However, possibly a SU(2) Higgs doublet has a mass much larger than $m_{\rm top}$. Then it is practically degenerate, and its approximate masses are given in the NMSSM by

$$M_{HD} = \lambda s(A_\lambda + \kappa s)(\cot\beta + \tan\beta)\,. \tag{C.8}$$

At scales below M_{HD} the RG equations read

$$\begin{aligned} \frac{dh_t}{d\ln Q^2} &= \frac{h_t}{64\pi^2}(9h_t^2 + 3h_b^2) - h_t\frac{\alpha_s}{\pi}\,, \\ \frac{dh_b}{d\ln Q^2} &= \frac{h_b}{64\pi^2}(9h_b^2 + 3h_t^2) - h_b\frac{\alpha_s}{\pi}\,. \end{aligned} \tag{C.9}$$

In order to correct the expressions for $h_t(Q)$, $h_b(Q)$ in this case, we have to multiply them by $\left(1 + \frac{h_b^2}{32\pi^2}\ln(M_{HD}^2/m_{\rm top}^2)\right)$, $\left(1 + \frac{h_t^2}{32\pi^2}\ln(M_{HD}^2/m_{\rm top}^2)\right)$, respectively, if $M_{HD} > m_{\rm top}$.

The expressions for the running top and bottom quark masses at the scale Q are then simply

$$m_{\rm top}(Q) = h_t(Q)h_u(Q)\,, \qquad m_{\rm bot}(Q) = h_b(Q)h_d(Q)\,, \tag{C.10}$$

and below it is convenient to use

$$\tan\beta(Q) = \frac{h_u(Q)}{h_d(Q)}\,. \tag{C.11}$$

[2]Here we use the $\overline{MS}$ relation between the pole mass and the running mass. In the $\overline{DR}$ scheme the second factor $4/3\pi$ would read $5/3\pi$, which would increase $h_t(m_{\rm top})$ and hence the lightest Higgs mass by $\sim 1\%$. Since we have not included subdominant (single) logarithms in the two loop corrections to the effective potential we are not sensitive to the scheme in which the running top quark mass is defined, which leads to a theoretical error of $\sim 1\%$ on the mass of the lightest Higgs.

For the (s)top, (s)bottom induced one loop corrections to V_{eff} we still need the eigenvalues of the stop, sbottom mass matrices (A.32) and (A.33). To this end it is convenient to define, step by step,

$$\begin{aligned}
M^2_{\rm stop} &= \frac{1}{2}(m_Q^2 + m_T^2)\,, \\
M^2_{\rm sbot} &= \frac{1}{2}(m_Q^2 + m_B^2)\,, \\
\Delta M^2_{\rm stop} &= \frac{1}{2}(m_Q^2 - m_T^2)\,, \\
\Delta M^2_{\rm sbot} &= \frac{1}{2}(m_Q^2 - m_B^2)\,, \\
X_t &= A_t - \lambda s \cot\beta(Q)\,, \\
X_b &= A_b - \lambda s \tan\beta(Q)\,, \\
W_t &= \sqrt{\Delta M^4_{\rm stop} + m^2_{\rm top}(Q) X_t^2}\,, \\
W_b &= \sqrt{\Delta M^4_{\rm sbot} + m^2_{\rm bot}(Q) X_b^2}\,,
\end{aligned} \tag{C.12}$$

with the help of which the eigenvalues of the stop, sbottom mass matrices become

$$\begin{aligned}
M^2_{\rm stop1} &= M^2_{\rm stop} + m^2_{\rm top}(Q) - W_t\,, \\
M^2_{\rm stop2} &= M^2_{\rm stop} + m^2_{\rm top}(Q) + W_t\,, \\
M^2_{\rm sbot1} &= M^2_{\rm sbot} + m^2_{\rm bot}(Q) - W_b\,, \\
M^2_{\rm sbot2} &= M^2_{\rm sbot} + m^2_{\rm bot}(Q) + W_b\,.
\end{aligned} \tag{C.13}$$

Finally the following auxiliary quantities are useful:

$$\begin{aligned}
fmt &= \frac{1}{2W_t}\left(M^2_{\rm stop2} \ln\left(\frac{M^2_{\rm stop2}}{Q^2}\right) - M^2_{\rm stop1} \ln\left(\frac{M^2_{\rm stop1}}{Q^2}\right)\right) - 1\,, \\
gmt &= \frac{m^2_{\rm top} X_t^2}{W_t^2}\left(\frac{M^2_{\rm stop2} + M^2_{\rm stop1}}{M^2_{\rm stop2} - M^2_{\rm stop1}} \ln\left(\frac{M^2_{\rm stop2}}{M^2_{\rm stop1}}\right) - 2\right)\,, \\
emt &= \frac{m^2_{\rm top} X_t}{W_t} \ln\left(\frac{M^2_{\rm stop2}}{M^2_{\rm stop1}}\right), \\
fmb &= \frac{1}{2W_b}\left(M^2_{\rm sbot2} \ln\left(\frac{M^2_{\rm sbot2}}{Q^2}\right) - M^2_{\rm sbot1} \ln\left(\frac{M^2_{\rm sbot1}}{Q^2}\right)\right) - 1\,, \\
gmb &= \frac{m^2_{\rm bot} X_b^2}{W_b^2}\left(\frac{M^2_{\rm sbot2} + M^2_{\rm sbot1}}{M^2_{\rm sbot2} - M^2_{\rm sbot1}} \ln\left(\frac{M^2_{\rm sbot2}}{M^2_{\rm sbot1}}\right) - 2\right)\,, \\
emb &= \frac{m^2_{\rm bot} X_b}{W_b} \ln\left(\frac{M^2_{\rm sbot2}}{M^2_{\rm sbot1}}\right),
\end{aligned} \tag{C.14}$$

and, remarkably, quite many of the radiative corrections can be absorbed into a shift of A_λ:

$$A_{\lambda,\rm new} = A_\lambda + \frac{3h_t(Q)^2}{16\pi^2} A_t\; fmt + \frac{3h_b(Q)^2}{16\pi^2} A_b\; fmb\,. \tag{C.15}$$

Subsequently the index new of $A_{\lambda,new}$ will be omitted.

Then the radiatively corrected mass-squared matrix entries in the CP even sector are, instead of (A.9),

$$
\begin{aligned}
\mathcal{M}^2_{S,11} &= g^2 h_u(Q)^2 + \lambda s(A_\lambda + \kappa s)\cot\beta(Q) - \frac{3h_b(Q)^2}{32\pi^2}(\lambda s)^2\ gmb + \\
&\quad + \frac{3h_t(Q)^2}{32\pi^2}\left(-A_t^2\ gmt + 4A_t\ emt + 4m^2_{\mathrm{top}}\ln\left(\frac{M^2_{\mathrm{stop1}}M^2_{\mathrm{stop2}}}{m^4_{\mathrm{top}}}\right)\right),\\
\mathcal{M}^2_{S,22} &= g^2 h_d(Q)^2 + \lambda s(A_\lambda + \kappa s)\tan\beta(Q) - \frac{3h_t(Q)^2}{32\pi^2}(\lambda s)^2\ gmt + \\
&\quad + \frac{3h_b(Q)^2}{32\pi^2}\left(-A_b^2\ gmt + 4A_b\ emb + 4m^2_{\mathrm{bot}}\ln\left(\frac{M^2_{\mathrm{sbot1}}M^2_{\mathrm{sbot2}}}{m^4_{\mathrm{bot}}}\right)\right),\\
\mathcal{M}^2_{S,33} &= \lambda A_\lambda \frac{h_u(Q)h_d(Q)}{s} + \kappa s(A_\kappa + 4\kappa s) - \\
&\quad - \frac{3h_t(Q)^2}{32\pi^2}\lambda^2 h_d(Q)^2\ gmt - \frac{3h_b(Q)^2}{32\pi^2}\lambda^2 h_u(Q)^2\ gmb\,,\\
\mathcal{M}^2_{S,12} &= (2\lambda^2 - g^2)h_u(Q)h_d(Q) - \lambda s(A_\lambda + \kappa s) + \\
&\quad + \lambda s\left(\frac{3h_t(Q)^2}{32\pi^2}(A_t\ gmt - 2\ emt) + \frac{3h_b(Q)^2}{32\pi^2}(A_b\ gmb - 2\ emb)\right),\\
\mathcal{M}^2_{S,13} &= 2\lambda^2 h_u(Q)s - \lambda h_d(Q)(A_\lambda + 2\kappa s) + \\
&\quad + \frac{3h_t(Q)^2}{32\pi^2}\lambda h_d(Q)(A_t\ gmt - 2\ emt) + \\
&\quad + \frac{3h_b(Q)^2}{32\pi^2}\lambda^2 s\ h_u(Q)(4\ fmb - gmb)\,,\\
\mathcal{M}^2_{S,23} &= 2\lambda^2 h_d(Q)s - \lambda h_u(Q)(A_\lambda + 2\kappa s) + \\
&\quad + \frac{3h_b(Q)^2}{32\pi^2}\lambda h_u(Q)(A_b\ gmb - 2\ emb) + \\
&\quad + \frac{3h_t(Q)^2}{32\pi^2}\lambda^2 s\ h_d(Q)(4\ fmt - gmt)\,.
\end{aligned}
\tag{C.16}
$$

The dominant two loop effects $\sim h_t^6 t^2$ and $\sim h_t^4 \alpha_s t^2$ contribute to $\mathcal{M}^2_{S,11}$ only. They read

$$
\delta\mathcal{M}^2_{S,11\ \mathrm{2loops}} = \frac{3h_t(Q)^4}{64\pi^4}h_u(Q)^2\left(32\pi\alpha_s(Q) - \frac{3}{2}h_t(Q)^2\right)t^2\,. \tag{C.17}
$$

The above expressions for the elements of the CP even mass matrix have been obtained by taking the second derivatives of the effective potential, and using the three minimization equations with respect to h_u, h_d and s in order to eliminate the three SUSY breaking masses squared. Since the Higgs fields are not quite properly normalized due to the Z factors, the above mass matrix elements still have to be rescaled by appropriate powers of Z_i (here below $\mathcal{M}^2_{S,ij\ norm.}$ denote the mass matrix elements for correctly normalized Higgs fields):

$$
\begin{aligned}
\mathcal{M}^2_{S,11\ \mathrm{norm.}} &= \frac{\mathcal{M}^2_{S,11}}{Z_{H_u}}\,,\\
\mathcal{M}^2_{S,22\ \mathrm{norm.}} &= \frac{\mathcal{M}^2_{S,22}}{Z_{H_d}}\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}^2_{S,12\ \text{norm.}} &= \frac{\mathcal{M}^2_{S,12}}{\sqrt{Z_{H_u} Z_{H_d}}}\,, \\
\mathcal{M}^2_{S,13\ \text{norm.}} &= \frac{\mathcal{M}^2_{S,13}}{\sqrt{Z_{H_u}}}\,, \\
\mathcal{M}^2_{S,23\ \text{norm.}} &= \frac{\mathcal{M}^2_{S,23}}{\sqrt{Z_{H_d}}}\,.
\end{aligned}
\tag{C.18}
$$

Now we turn to the electroweak leading logarithmic corrections to the Higgs mass matrix elements, which are taken from ref. [15]. As in [15] we allow for a Higgs doublet which is possibly heavy ($M_{HD} > M_Z$), and for gauginos/Higgsinos with masses possibly (much) larger than M_Z. In this case their masses are given approximately by the soft SUSY breaking gaugino mass parameters M_1, M_2 or $\mu_{\text{eff}} = \lambda s$, respectively. The following auxiliary parameters are useful subsequently:

$$
\begin{aligned}
\mu_1 &= \max\{|\mu_{\text{eff}}|,\ M_1\}\,, \\
\mu_2 &= \max\{|\mu_{\text{eff}}|,\ M_2\}\,, \\
\mu_{12} &= \max\{|\mu_{\text{eff}}|,\ M_1,\ M_2\}\,.
\end{aligned}
\tag{C.19}
$$

The electroweak corrections to the Higgs mass matrix elements are then given by (we omit the index norm. in the following)

$$
\begin{aligned}
&\delta_{ew}\mathcal{M}^2_{S,11} = \\
&= \frac{g_2^2 M_Z^4}{16\pi^2 M_W^2}\sin^2\beta\Bigg(\left(4 - 8\sin^2\theta_W + \frac{32}{3}\sin^4\theta_W\right) t + \\
&\qquad + \frac{1}{6}\Big[-9(\cos^2\beta - \sin^2\beta)^4 + \\
&\qquad\qquad + (1 - 2\sin^2\theta_W + 2\sin^4\theta_W)(\cos^2\beta - \sin^2\beta)^2 + \\
&\qquad\qquad + 10 - 2\sin^2\theta_W + 2\sin^4\theta_W\Big] \ln\left(\frac{M_{HD}^2}{M_Z^2}\right) + \\
&\qquad + \sin^2\theta_W(1 - 2\sin^2\theta_W)\ln\left(\frac{\mu_1^2}{M_Z^2}\right) - \\
&\qquad - 4\sin^2\theta_W\cos^2\theta_W\ln\left(\frac{\mu_{12}^2}{M_Z^2}\right) + \\
&\qquad + \cos^2\theta_W(3 - 10\cos^2\theta_W)\ln\left(\frac{\mu_2^2}{M_Z^2}\right) - \frac{4}{3}\cos^4\theta_W \ln\left(\frac{M_2^2}{M_Z^2}\right) - \\
&\qquad - \frac{2}{3}(\sin^2\theta_W + \cos^4\theta_W)\ln\left(\frac{\mu_{\text{eff}}^2}{M_Z^2}\right)\Bigg)\,,
\end{aligned}
$$

$$
\begin{aligned}
&\delta_{ew}\mathcal{M}^2_{S,22} = \\
&= \frac{g_2^2 M_Z^4}{16\pi^2 M_W^2}\cos^2\beta\Bigg(\left(4 - 8\sin^2\theta_W + \frac{32}{3}\sin^4\theta_W\right) t +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{6}\Big[-9(\cos^2\beta-\sin^2\beta)^4+\\
&\quad+(1-2\sin^2\theta_W+2\sin^4\theta_W)(\cos^2\beta-\sin^2\beta)^2+\\
&\quad+10-2\sin^2\theta_W+2\sin^4\theta_W\Big]\ln\left(\frac{M_{HD}^2}{M_Z^2}\right)+\\
&+\sin^2\theta_W(1-2\sin^2\theta_W)\ln\left(\frac{\mu_1^2}{M_Z^2}\right)-4\sin^2\theta_W\cos^2\theta_W\ln\left(\frac{\mu_{12}^2}{M_Z^2}\right)+\\
&+\cos^2\theta_W(3-10\cos^2\theta_W)\ln\left(\frac{\mu_2^2}{M_Z^2}\right)-\frac{4}{3}\cos^4\theta_W\ln\left(\frac{M_2^2}{M_Z^2}\right)-\\
&-\frac{2}{3}(\sin^2\theta_W+\cos^4\theta_W)\ln\left(\frac{\mu_{\text{eff}}^2}{M_Z^2}\right)\Bigg),
\end{aligned}
$$

$$
\begin{aligned}
&\delta_{ew}\mathcal{M}_{S,12}^2=\\
&=\frac{g_2^2M_Z^4}{16\pi^2M_W^2}\sin\beta\cos\beta\Bigg(\left(-4+8\sin^2\theta_W-\frac{32}{3}\sin^4\theta_W\right)t-\\
&\qquad-\frac{1}{6}[-9(\cos^2\beta-\sin^2\beta)^4+\\
&\qquad\quad+(1-2\sin^2\theta_W+2\sin^4\theta_W)(\cos^2\beta-\sin^2\beta)^2+\\
&\qquad\quad+8-22\sin^2\theta_W+10\sin^4\theta_W]\ln\left(\frac{M_{HD}^2}{M_Z^2}\right)-\\
&\qquad-\sin^2\theta_W(1+2\sin^2\theta_W)\ln\left(\frac{\mu_1^2}{M_Z^2}\right)-\\
&\qquad-4\sin^2\theta_W\cos^2\theta_W\ln\left(\frac{\mu_{12}^2}{M_Z^2}\right)-\\
&\qquad-\cos^2\theta_W(3+2\cos^2\theta_W)\ln\left(\frac{\mu_2^2}{M_Z^2}\right)+\frac{4}{3}\cos^4\theta_W\ln\left(\frac{M_2^2}{M_Z^2}\right)+\\
&\qquad+\frac{2}{3}(\sin^2\theta_W+\cos^4\theta_W)\ln\left(\frac{\mu_{\text{eff}}^2}{M_Z^2}\right)\Bigg).
\end{aligned}
\tag{C.20}
$$

The eigenvalues of this Higgs mass matrix give the "running" Higgs masses at a scale m_{top} due to the argument of the logarithm t in the Z factors (C.1). In the LLA we can identify the scales m_{top}, M_Z (in the arguments of the logarithms) and, to start with, M_{Higgs}, and take these "running" masses as pole masses. However, if a Higgs mass is (much) larger than m_{top}, we should have computed the Z factors (C.1) at a scale M_{Higgs} rather than m_{top}, in order to identify the eigenvalues of the Higgs mass matrix with the Higgs pole masses. We can correct for this at the end (after having diagonalized the Higgs mass matrix), taking care that now the eigenstates h_i are given by $h_i = S_{ij}S_j^{bare}$, cf. the notation below eq. (A.9). Hence, if $M_{\text{Higgs}} > m_{\text{top}}$, the Higgs pole masses $M_i^2(pole)$ are given in terms of the eigenvalues M_i^2 as

$$
M_i^2(\text{pole}) = \frac{M_i^2}{\left(1-\left(\frac{3h_t^2}{16\pi^2}S_{i1}^2+\frac{3h_b^2}{16\pi^2}S_{i2}^2\right)\ln\left(M_i^2/m_{\text{top}}^2\right)\right)}. \tag{C.21}
$$

Now we turn to the radiative corrections to the CP odd mass matrix. After the shift (C.15) in A_λ these are trivial: It suffices to take the tree level expressions (A.11) for the mass matrix elements, to replace the Higgs vevs h_u and h_d by its expressions (C.3) at the scale Q, and to rescale the mass matrix elements by appropriate powers of Z_i as in (C.18) for the CP even sector. After dropping the Goldstone mode this gives a 2×2 mass matrix (instead of (A.13))

$$\begin{aligned}
\mathcal{M}^2_{P,11} &= \lambda s(A_\lambda + \kappa s)\left(\frac{\tan\beta(Q)}{Z_{H_d}} + \frac{\cot\beta(Q)}{Z_{H_u}}\right), \\
\mathcal{M}^2_{P,22} &= 4\lambda\kappa h_u(Q)h_d(Q) + \lambda A_\lambda \frac{h_u(Q)h_d(Q)}{s} - 3\kappa A_\kappa s\,, \\
\mathcal{M}^2_{P,12} &= \lambda\sqrt{h_u(Q)^2/Z_{H_d} + h_d(Q)^2/Z_{H_u}}\,(A_\lambda - 2\kappa s)\,.
\end{aligned} \tag{C.22}$$

Finally, if a CP odd Higgs mass is (much) larger than $m_{\rm top}$, its pole mass is given by an expression analogous to (C.21) after replacing S_{i1} by $P'_{i1}\cos^2\beta$ and S_{i2} by $P'_{i1}\sin^2\beta$ (see (A.14) for the definition of P'_{ij}).

In the expression for radiatively corrected charged Higgs mass we neglect stop/sbottom loops (without large logarithms), but we include large logarithms proportional to electroweak gauge couplings:

$$\begin{aligned}
M^2_{H^\pm} = &\left(\lambda s(A_\lambda + \kappa s) + h_u(Q)h_d(Q)\left(\frac{g_2^2}{2} - \lambda^2\right)\right)\left(\frac{\tan\beta(Q)}{Z_{H_d}} + \frac{\cot\beta(Q)}{Z_{H_u}}\right) + \\
&+ \frac{3h_t^2h_b^2 t}{(16\sqrt{2}\pi^2 G_F)} + \\
&+ \frac{g_2^2 M_W^2}{48\pi^2}\left(12t + 3\tan^2\theta_W\left(\ln\left(\frac{\mu_1^2}{M_Z^2}\right) + 4\ln\left(\frac{\mu_{12}^2}{M_Z^2}\right)\right) - \right. \\
&\left. \qquad - 3\ln\left(\frac{\mu_2^2}{M_Z^2}\right) - 4\ln\left(\frac{M_2^2}{M_Z^2}\right) - 2\ln\left(\frac{\mu_{\rm eff}^2}{M_Z^2}\right)\right).
\end{aligned} \tag{C.23}$$

This time, once $M_{H^\pm}$ is larger than $m_{\rm top}$, its pole mass is given by

$$M^2_{H^\pm}(pole) = \frac{M^2_{H^\pm}}{\left(1 - \left(\frac{3h_t^2}{16\pi^2}\cos^2\beta + \frac{3h_b^2}{16\pi^2}\sin^2\beta\right)\ln\left(M^2_{H^\pm}/m^2_{\rm top}\right)\right)}\,. \tag{C.24}$$

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